Block-Wise MAP Inference for Determinantal Point Processes with Application to Change-Point Detection

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Background

Determinantal Point Processes (DPPs)

- A probability measure over subsets of a point set

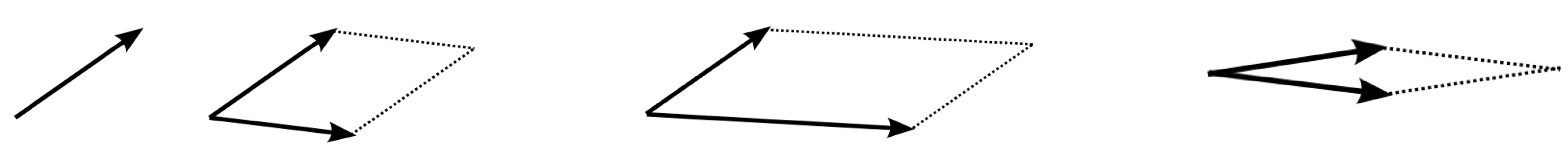
$$\mathcal{P}_{\mathbf{L}}(Y) \propto \det(\mathbf{L}_Y),\ \forall\, Y \subset \mathcal{Y} = \{1, ..., N\}$$

- Quality-diversity decomposition of L-ensemble kernel :

$$\mathbf{L} = diag(\mathbf{q})\ \mathbf{S}\ diag(\mathbf{q}),$$

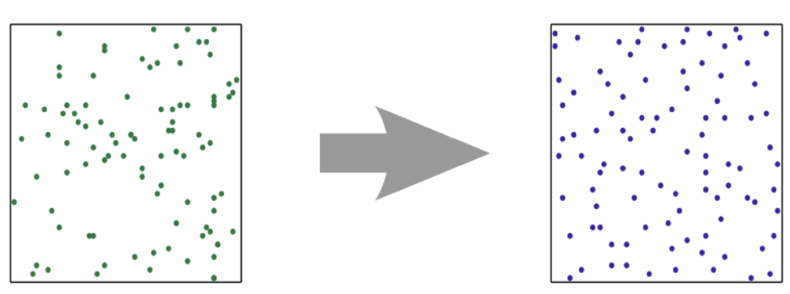
$\mathbf{q} \in \mathbb{R}^N$: quality of each item; $\mathbf{S} \in \mathbb{R}^{N\times N}$: similarity of each item pair.

- DPPs encourage both quality and diversity:
  - For items i, j , $\mathcal{P}_{\mathbf{L}}(\{i,j\})$ is large if q_i, q_j are large and S_{ij} is small.



Magnitude: quality. **Direction**: similarity. **Area**: probability.

- DPP sample:



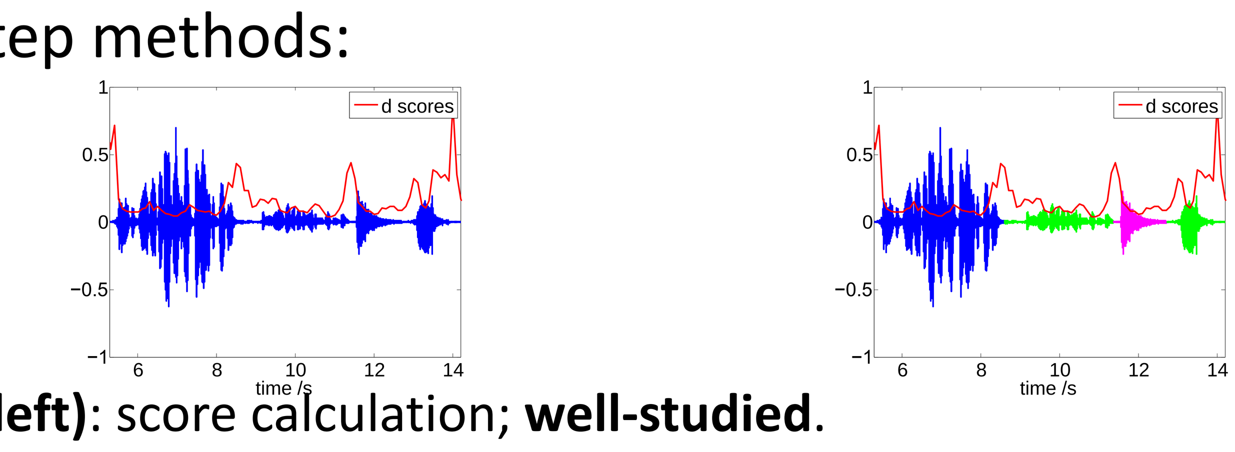
Random sample   DPP sample

Maximum a posterior problem of DPP (MAP)

- Find the most probable subset.
- **NP-hard**. Approximate algorithms require at least $O(N^{2.4})$.

Change-point Detection Problem (CPD)

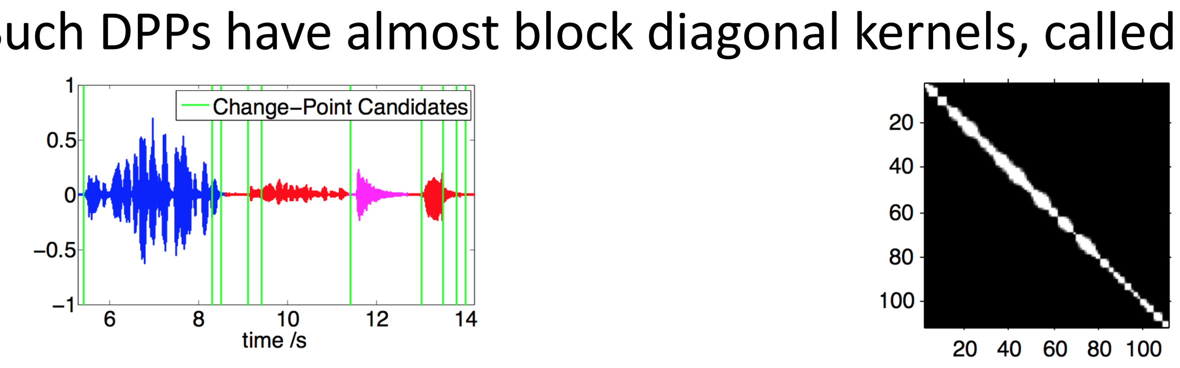
- Two-step methods:



Step 1 (left): score calculation; **well-studied**.
Step 2 (right): change points selection; **lack of study.**

Contributions

- A new CPD method: **BwDppCpd (Step 2 by DPP MAP)**
  - Such DPPs have almost block diagonal kernels, called BwDPPs.



States do not change rapidly !

CPD of a speech signal   The corresponding BwDPP

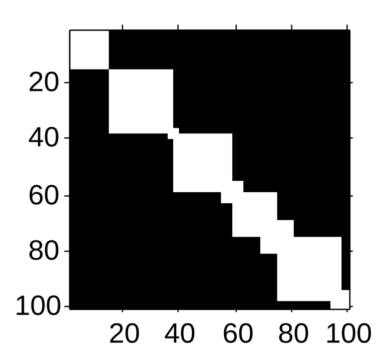
- An $O(N)$ DPP MAP method for such kernel: **BwDPP-MAP**

Methodology

BwDPPs: DPPs with almost block diagonal kernels

- Almost block diagonal kernel

$$\mathbf{L} \triangleq \begin{bmatrix} \mathbf{L}_1 & \mathbf{A}_1 & & \cdots & 0 \\ \mathbf{A}_1^T & \mathbf{L}_2 & \mathbf{A}_2 & & \\ & \ddots & \ddots & \ddots & \vdots \\ & & \mathbf{A}_{m-2}^T & \mathbf{L}_{m-1} & \mathbf{A}_{m-1} \\ 0 & \cdots & & \mathbf{A}_{m-1}^T & \mathbf{L}_m \end{bmatrix}$$



- $\mathbf{L}_i$: diagonal components, dense;
- $\mathbf{A}_i$: off-diagonal components; non-zero only in bottom left.

BwDPP-MAP: Fast MAP Inference for BwDPPs

- Method: for $i = 1, ..., m$,

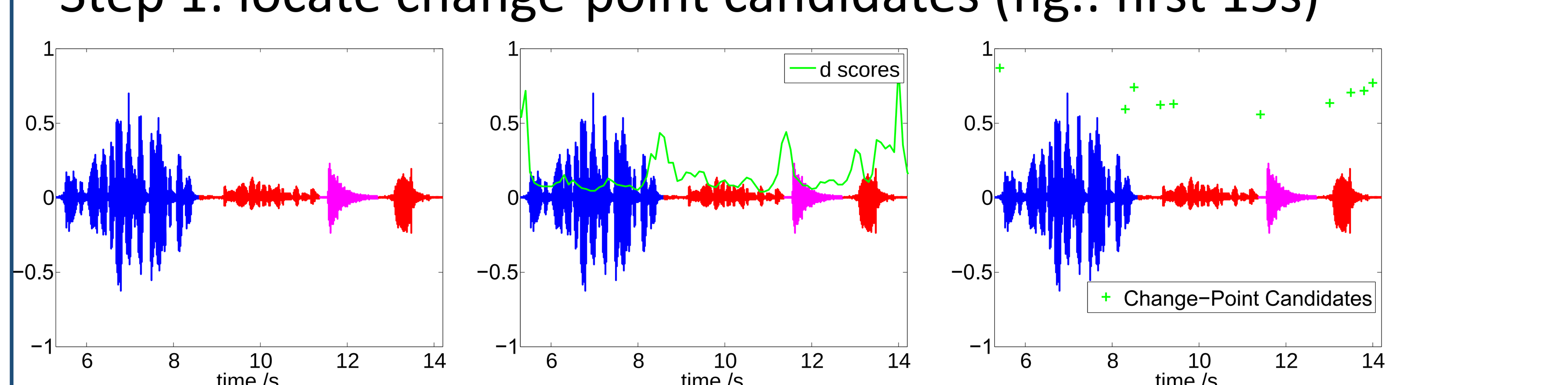
Conditioning: $\mathbf{L}_i \rightarrow \tilde{\mathbf{L}}_i$ ➔ Sub-inference: $\hat{C}_i = MAP(\tilde{\mathbf{L}}_i)$

- $\tilde{\mathbf{L}}_i$: kernel conditional on previous selection.
- $MAP()$: any DPP MAP algorithm.

- Highlights:
  - $O(N)$ complexity
  - A universal speed booster for **any** DPP-MAP algorithm

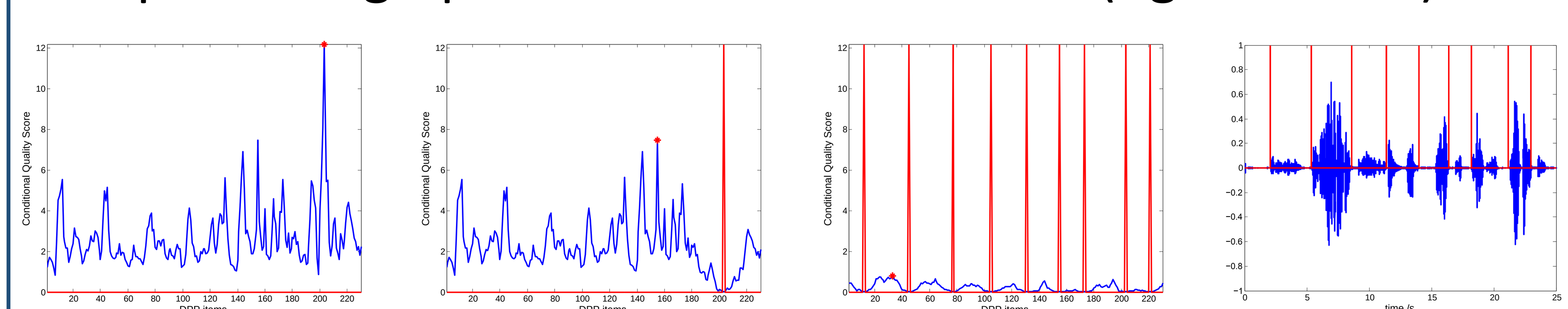
BwDppCpd: BwDPP-based Change-Point Detection

- Step 1: locate change-point candidates (fig.: first 15s)



Speech signal   Score calculation   Change-point candidate selection

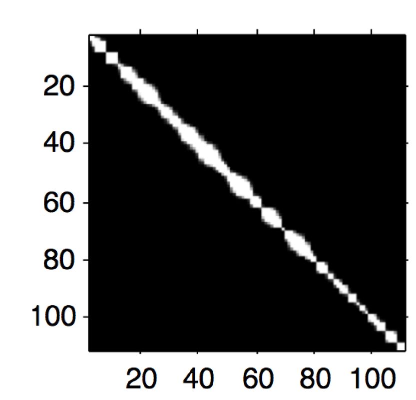
- Step 2: change-point selection via BwDPP (fig.: first 25s)



(a) (b) (c) (d)

(a) Quality of all candidates. **(b)** The candidates' neighbors' quality are reduced. **(c) & (d)** Final segmentation result.

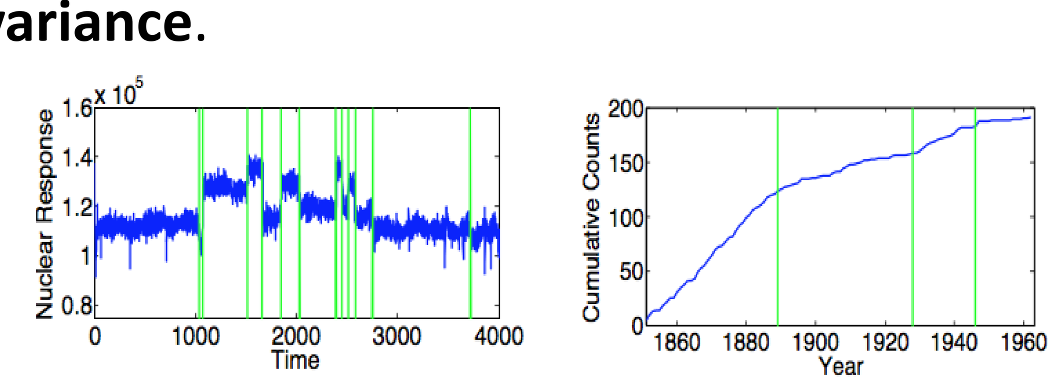
- BwDPP: modeling (fig.: 120s)
  - Quality: the dissimilarity score -> $\mathbf{q}$
  - Diversity: $S_{ij} \triangleq exp\left\{-\left(t_i - t_j\right)^2/\sigma^2\right\}$



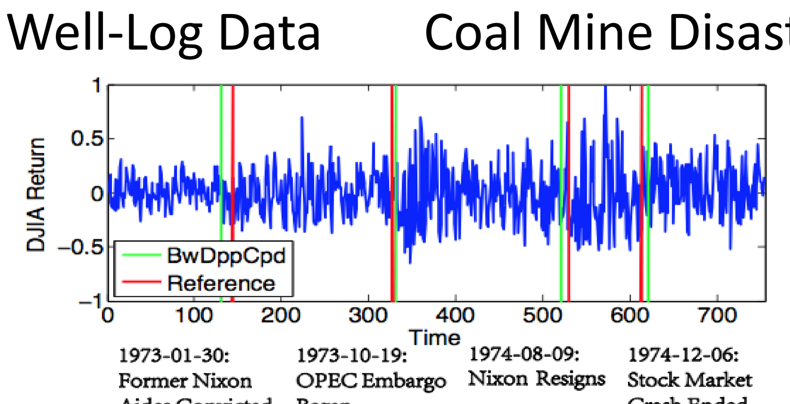
Experiments

Classic CPD Testing Datasets : kernel size ~ 100

- **Well-Log Data: varying Gaussian mean.**
- **Coal Mine Disaster Data: varying Poisson rate.**
- **1972-75 Dow Jones Industrial Average Return (DJIA): varying Gaussian variance.**



Well-Log Data   Coal Mine Disaster Data



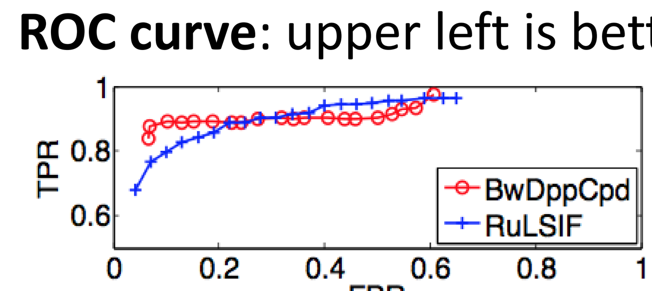
1972-75 DJIA

Human Activity Detection (HASC): kernel size ~1000

- Detecting human activity changes.
- Reference: RuLSIF (best) .

	PRC%	RCL%	F_1
BwDppCpd	93.05	87.88	0.9039
RuLSIF	86.36	83.84	0.8508

ROC curve: upper left is better.



Speech Segmentation

- Datasets
  - **Hub4m97**, Mandarin Broadcast News Speech.
  - **TelRecord**, telephone conversations.
- Reference : **DistBIC**.

	DistBIC	Bw-0	Bw-2
Hub4m97			
PRC%	64.29	65.29	65.12
RCL%	74.98	78.49	78.39
F_1	0.6922	0.7128	0.7114
TelRecord			
PRC%	61.39	66.54	66.47
RCL%	81.72	85.47	84.83
F_1	0.7011	0.7483	0.7454