

# LEARNING SPARSE STRUCTURED ENSEMBLES WITH STOCHASTIC GRADIENT MCMC SAMPLING AND NETWORK PRUNING

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# Outline

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- Motivation & Problem
- Related Work
- Our solution: Mix of multiple ingredients
  - Learning ensembles via SG-MCMC sampling
  - Cost reduction via structured model compression
  - Experimental results
- Conclusion & Future Work

# Ensemble of Neural Networks

- Ensemble models are a **group of models** that work collectively to get the averaged prediction.

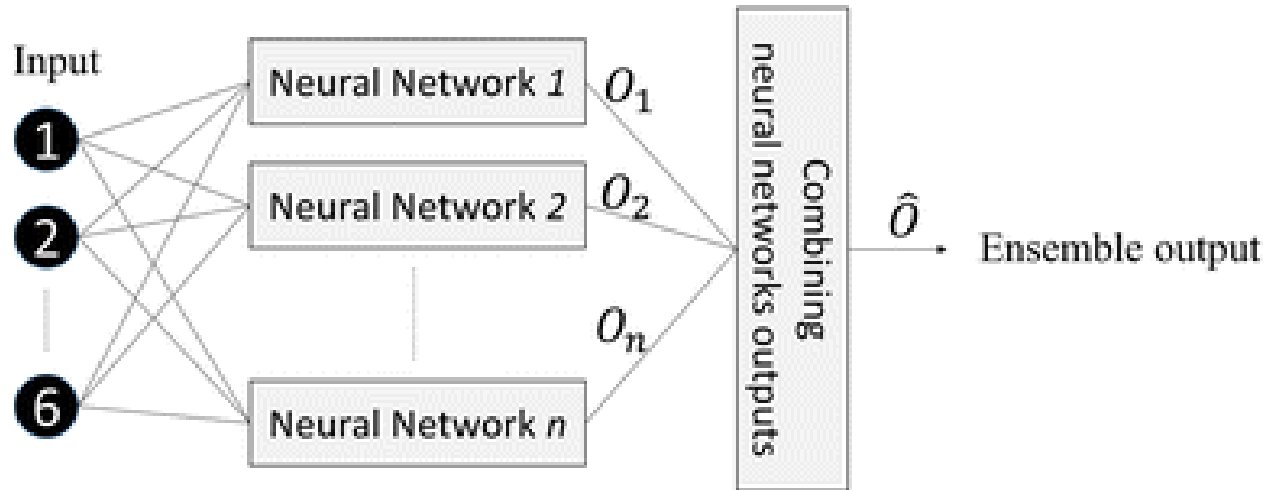


Figure from Effat Dehghanian et al. 2015

# Ensemble of Neural Networks

- Ensemble gives a **great boost in accuracy** because it does not rely on a single model for prediction.

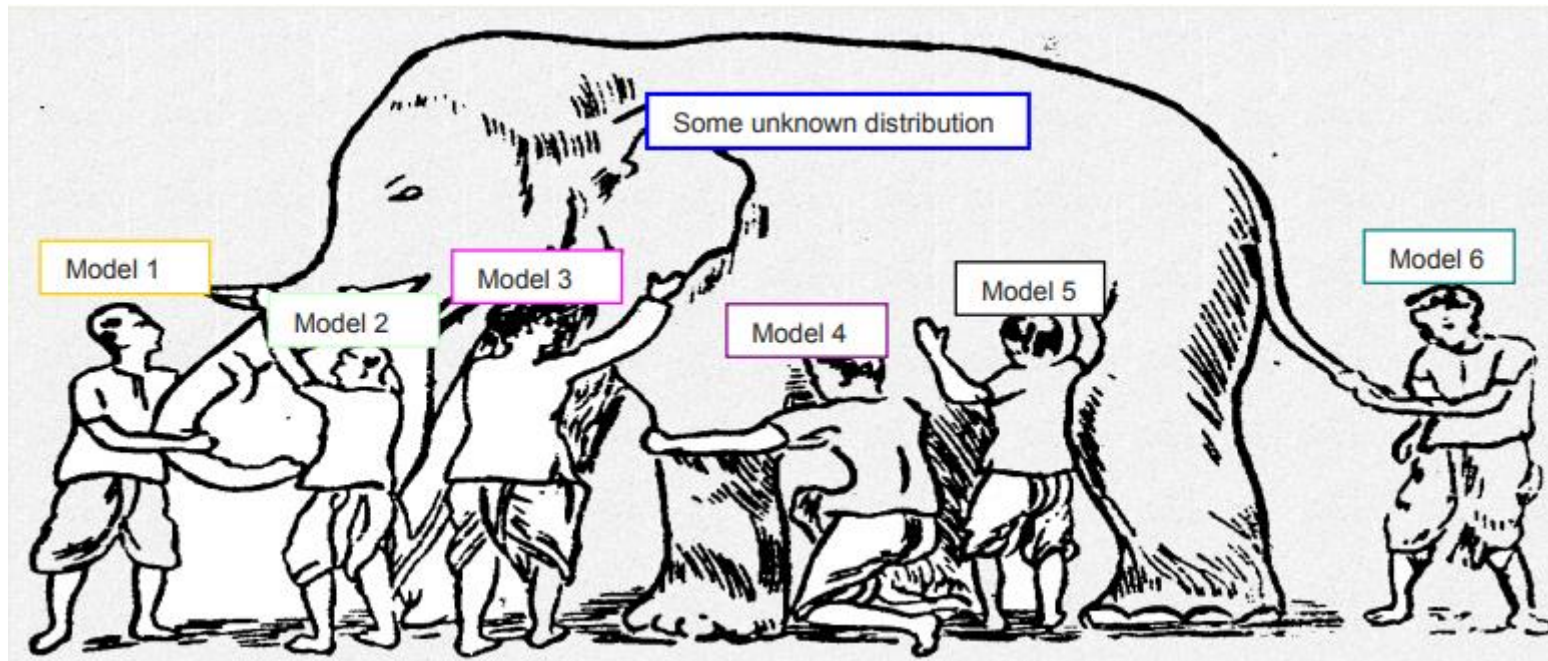


Figure from Alejandro Correa et al. 2013

Ordered by classification error

Team name	Entry description
WMW	Ensemble C [No bounding box results]
WMW	Ensemble E [No bounding box results]
WMW	Ensemble A [No bounding box results]
WMW	Ensemble D [No bounding box results]
WMW	Ensemble B [No bounding box results]
Trimps-Soushen	Result-1
Trimps-Soushen	Result-2
Trimps-Soushen	Result-3
Trimps-Soushen	Result-4
Trimps-Soushen	Result-5
NUS-Qihoo_DPNs (CLS-LOC)	[E2] CLS:: D

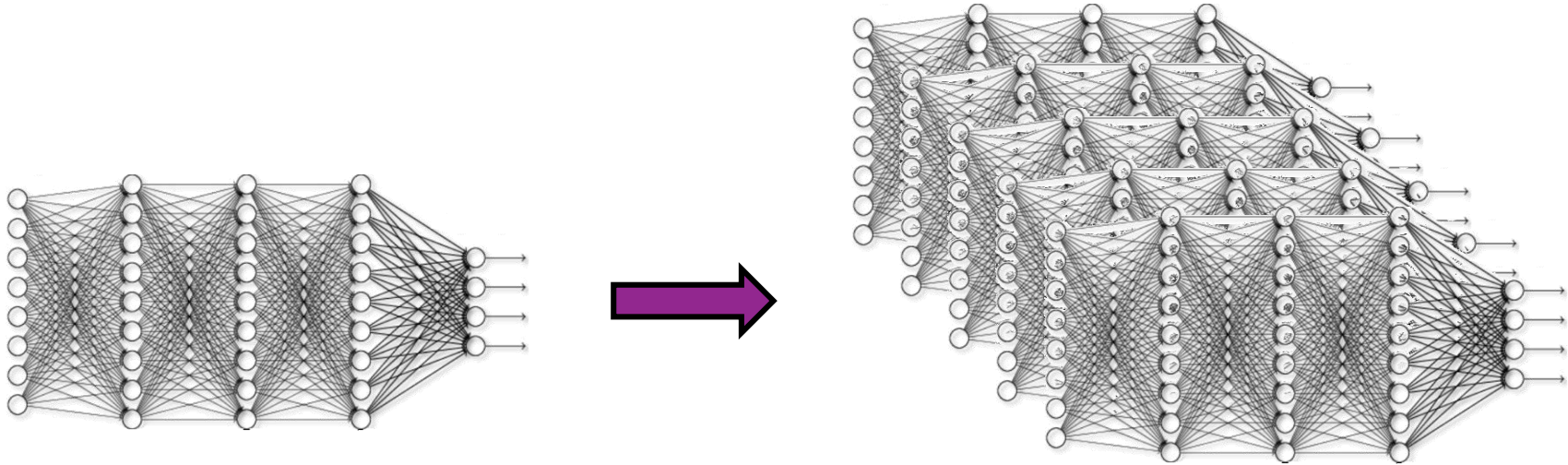
WER test-clean	Paper	Published	Notes
5.83%	Deep Speech 2: End-to-End Speech Recognition in English and Mandarin	December 2015	Humans
3.19%	The CAPIO 2017 Conversational Speech Recognition System	April 2018	TDNN + TDNN-LSTM + CNN-bLSTM + Dense TDNN-LSTM across two kinds of trees
2.22%	Improved training of end-to-end attention models for ASR	Interspeech, Sept 2018	encoder-attention-decoder end-to-end model

Method	test BLEU score (ntst14)
Bahdanau et al. [2]	28.45
Baseline System [29]	33.30
Single forward LSTM, beam size 12	26.17
Single reversed LSTM, beam size 12	30.59
Ensemble of 5 reversed LSTMs, beam size 1	33.00
Ensemble of 5 reversed LSTMs, beam size 12	33.27
Ensemble of 5 reversed LSTMs, beam size 2	34.50
Ensemble of 5 reversed LSTMs, beam size 12	<b>34.81</b>

CLASSIFIER	
large conv. net, unsup featurization	0.60
large conv. net, unsup pretraining [no distortions]	0.39
large conv. net, unsup pretraining [elastic distortions]	0.53
large/deep conv. net, 1-20-40-60-80-100-120-120-10 [elastic distortions]	0.35
committee of 7 conv. net, 1-20-P-40-P-150-10 [elastic distortions]	0.27 +-0.02
committee of 35 conv. net, 1-20-P-40-P-150-10 [elastic distortions]	0.23

Model	EM	F1
Human Performance Stanford University (Rajpurkar et al. '16)	82.304	91.221
MARS (ensemble)	83.982	89.796
ANFUDAO research NLP		
QANet (ensemble)	83.877	89.737
Google Brain & CMU		
Hybrid AoA Reader (ensemble)	82.482	89.281
Laboratory of HIT and iFLYTEK Research		
MARS (single model)	83.122	89.224
YUANFUDAO research NLP		
QANet (single)	82.471	89.306
Google Brain & CMU		
QANet (ensemble)	82.744	89.045
Google Brain & CMU		

# Problems

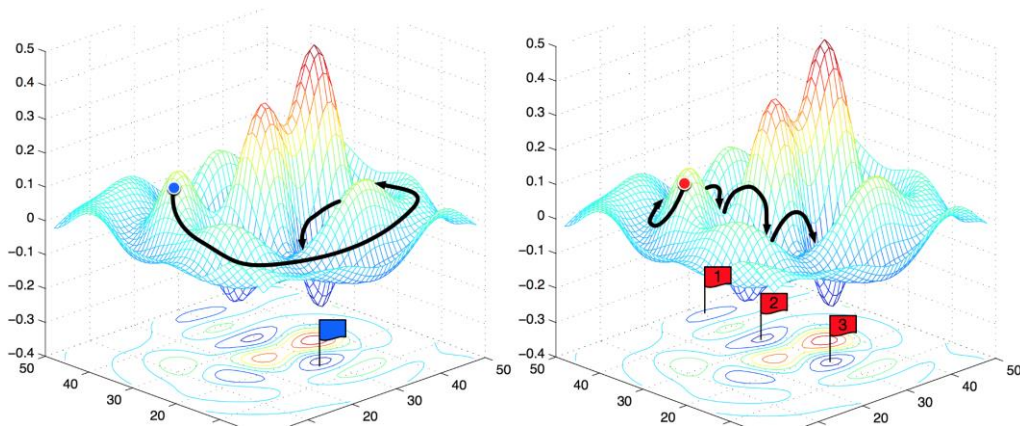
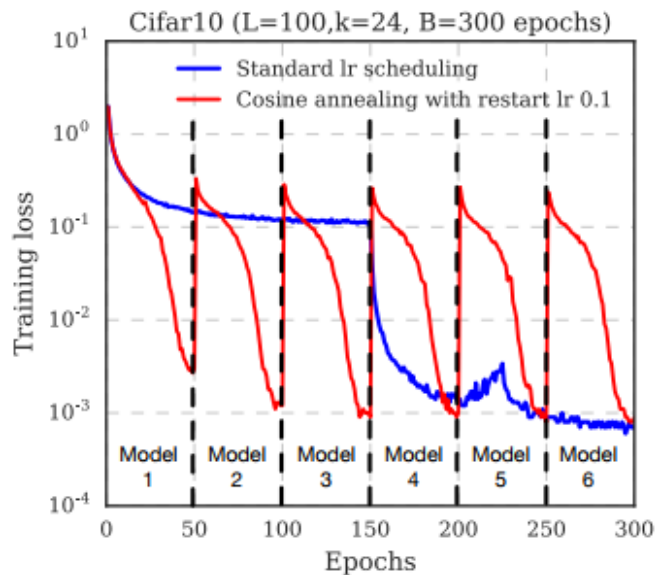


- **Training problem:**  $N$  times training time
- **Testing problem:**  $N$  times memory/testing time cost

# Related Work

## Snapshot ensembles: Train 1, get m for free (Gao Huang et al. 2017)

- Obtain multiple snapshot models **within a single training process**.
- Empirical cyclic learning rate settings.



## Related Work

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- The recent progress in Bayesian posterior sampling:  
**Stochastic Gradient Markov Chain Monte Carlo sampling** algorithms  
(Max Welling et al. 2011, Tianqi Chen et al. 2014, Zhe Gan et al. 2016)
- SG-MCMC works by **adding a scaled gradient noise** to Stochastic optimization method which is proved to have the following benefits :
  - (i) Theoretically interpretable
  - (ii) Efficient exploration of the model parameter space
  - (iii) Scalable and simple



# Related Work

- Testing problem:  $N$  times memory/testing time cost

**Model compression** via  
Network pruning and retraining  
(Song Han et al. 2015, 2017).

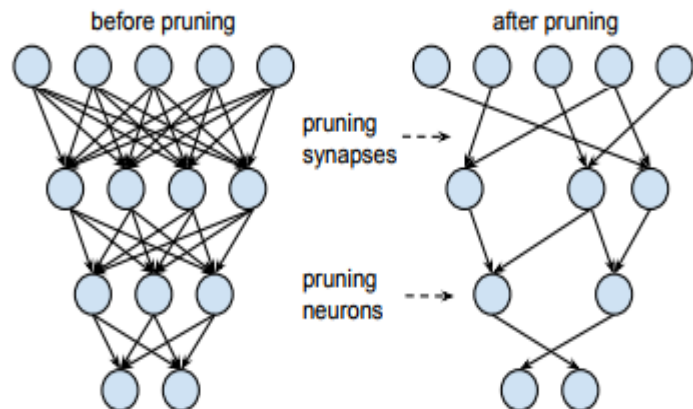


Figure from Song Han et al. 2015

**Sparse structure learning** via  
Group Lasso penalty (Ming Yuan et al. 2006)  
on deep models (Wei Wen et al. 2016, 2017).

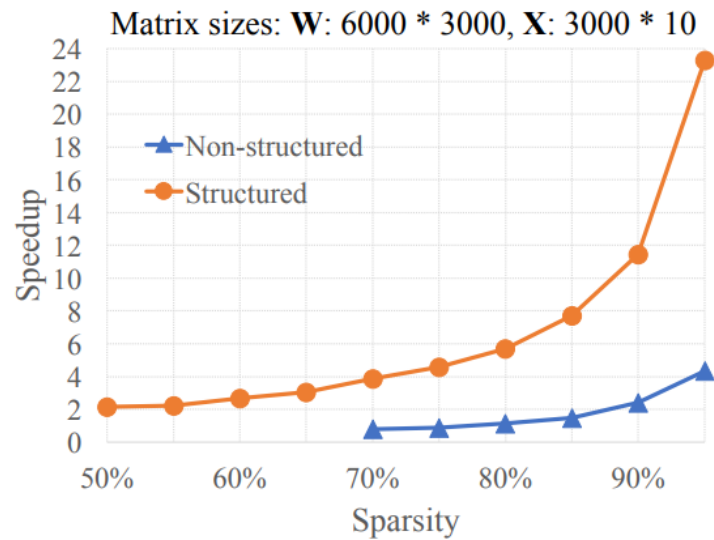


Figure from Wei Wen et al. 2017

# Our Propose

SG-MCMC based  
Bayesian learning



Group Sparse Prior



Network Pruning  
& Retraining

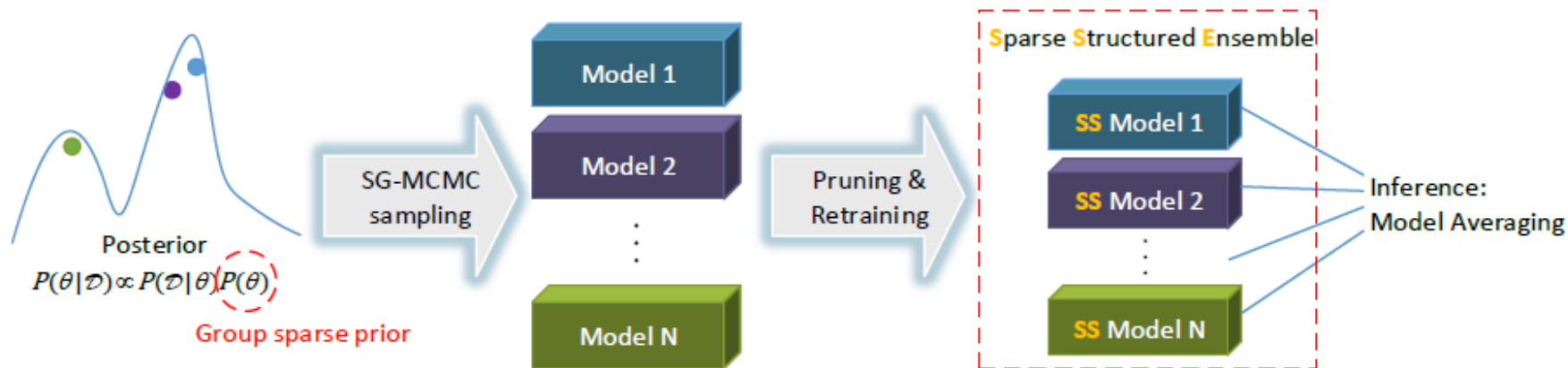


Figure 1: Overview of our two-stage method for learning SSEs.

# Bayesian Neural Network Framework

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- Denote  $\theta$  as all the trainable parameters in a neural network.
- Given data  $D = \{(x_i, y_i)\}_{i=1}^N$ , where input  $x_i \in \mathbb{R}^D$  and label  $y_i \in \mathcal{Y}$
- The goal of training is to evaluate the posterior distribution:

$$p(\theta|D) \propto p(\theta) \prod_{i=1}^N p(y_i|x_i, \theta) \quad (1)$$

- Given a testing input  $\tilde{x}$ , the Bayesian predictive distribution

$$p(\tilde{y}|\tilde{x}, D) = \mathbb{E}_{p(\theta|D)}[p(\tilde{y}|\tilde{x}, \theta)] = \int_{\theta} p(\tilde{y}|\tilde{x}, \theta)p(\theta|D)d\theta \quad (2)$$

$$p(\tilde{y}|\tilde{x}, D) \approx \frac{1}{M} \sum_{m=1}^M p(\tilde{y}|\tilde{x}, \theta_m) \quad , \theta_m \sim p(\theta|D) \quad (3)$$

can be considered as the average of NN softmax outputs.

# Training: SG-MCMC Sampling

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- Goal: sample  $\theta \sim p(\theta|D)$ , obtain  $\{\theta_m\}_{m=1}^M$
- Method: Stochastic Gradient Markov Chain Monte Carlo (SG-MCMC)

Stochastic Gradient Descent:

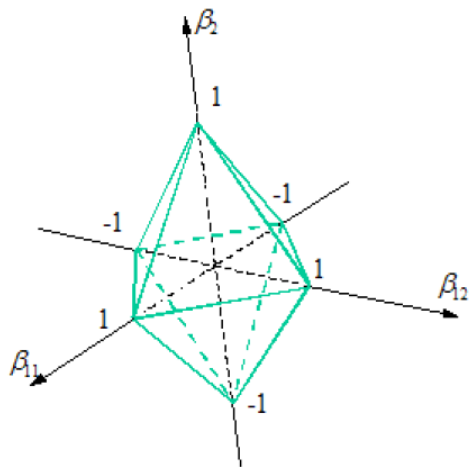
$$\tilde{g}_t = \frac{N}{n} \sum_{i=1}^n \nabla \log p(y_t^{(i)} | x_t^{(i)}, \theta_t),$$
$$\Delta\theta_t = \epsilon_t \tilde{g}_t$$

**Stochastic Gradient Langevin Dynamic** (Max Welling and Yee W Teh, 2011):

$$\tilde{g}_t = \nabla \log p(\theta_t) + \frac{N}{n} \sum_{i=1}^n \nabla \log p(y_t^{(i)} | x_t^{(i)}, \theta_t),$$
$$\Delta\theta_t = \epsilon_t \tilde{g}_t + \eta_t, \quad \eta_t \sim \mathcal{N}(0, 2\epsilon_t)$$

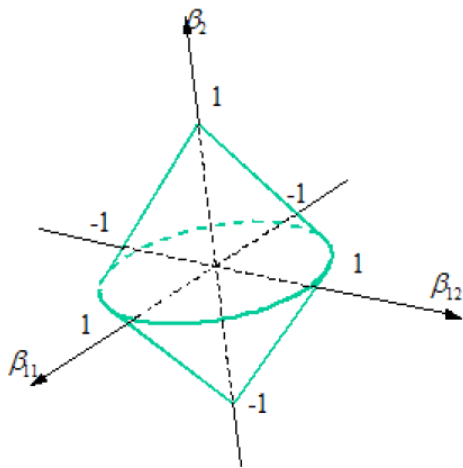
# Group Sparse Prior

$L_1$ : Random sparsity



$$\begin{aligned}\beta_{11} &= 0 \text{ or} \\ \beta_{12} &= 0 \text{ or} \\ \beta_2 &= 0\end{aligned}$$

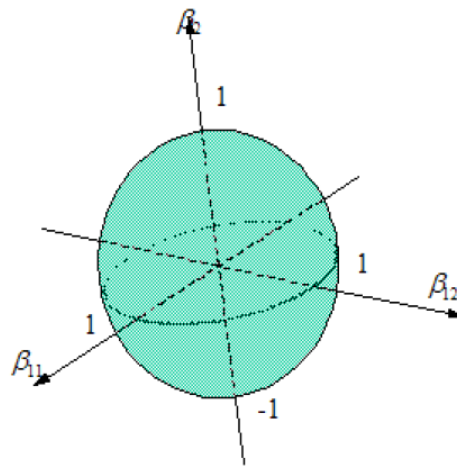
$L_{21}$ : Group sparsity



$$\begin{aligned}(\beta_{11}, \beta_{12}) &= 0 \text{ or} \\ \beta_2 &= 0\end{aligned}$$

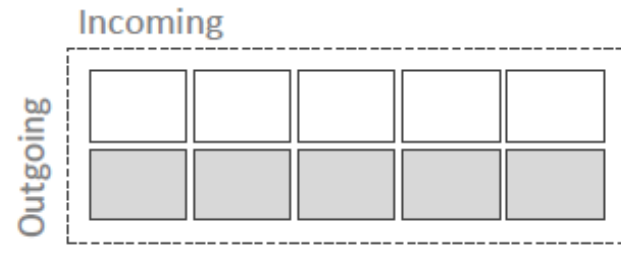
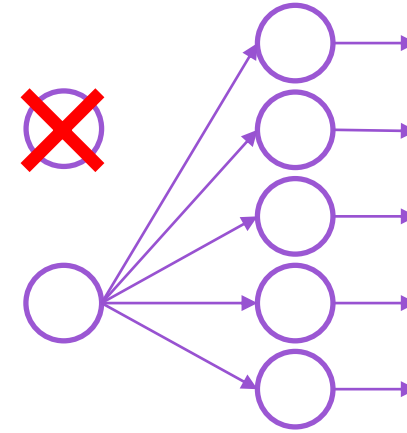
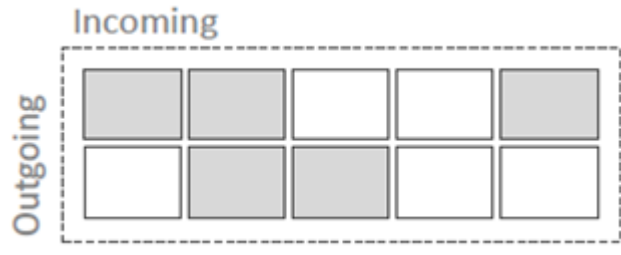
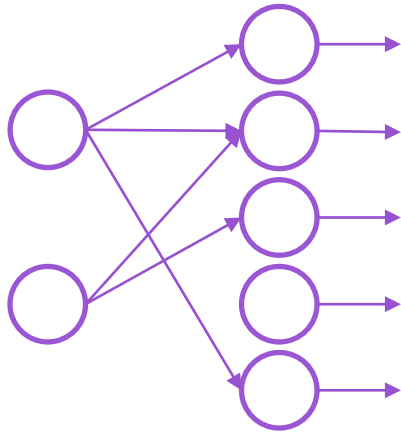
**group sparse prior**

$L_2$ : no sparsity



# Sparse Structured FNN

- Pruning of Fully-connected Neural Networks



# Sparse Structured LSTM

- Pruning of LSTMs

$$f_t = \sigma([\mathbf{x}_t, \mathbf{h}_{t-1}]W_f + \mathbf{b}_f)$$

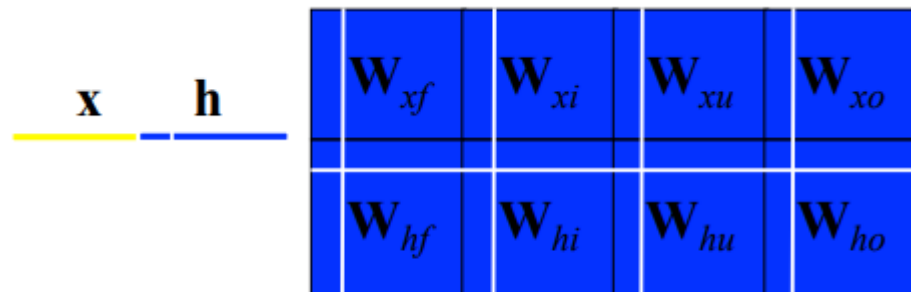
$$u_t = \tanh([\mathbf{x}_t, \mathbf{h}_{t-1}]W_u + \mathbf{b}_c)$$

$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + i_t \odot u_t$$

$$i_t = \sigma([\mathbf{x}_t, \mathbf{h}_{t-1}]W_i + \mathbf{b}_i)$$

$$o_t = \sigma([\mathbf{x}_t, \mathbf{h}_{t-1}]W_o + \mathbf{b}_o)$$

$$\mathbf{h}_t = o_t \odot \tanh(\mathbf{c}_t)$$



Weights matrices in LSTM

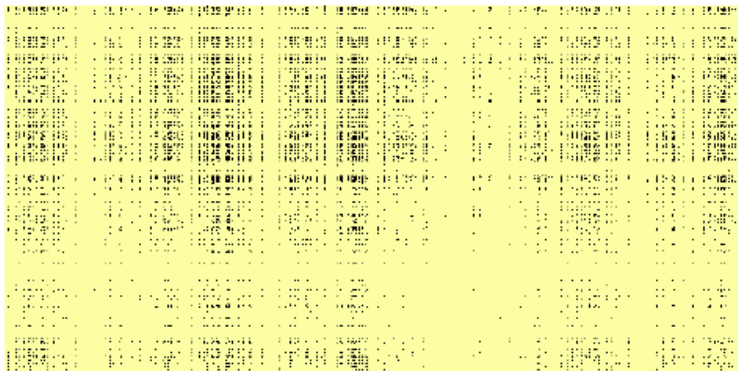


Weights in next layer(s)

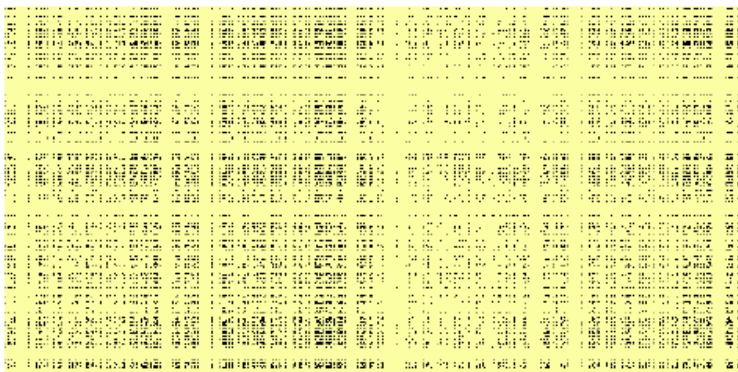
Figure from Wei Wen et al. 2017

# Sparse Structured LSTM

LSTM layer 1



LSTM layer 2



	Structure	Params	FLOPs
Original model	1500-1500-1500	51M	100%
Pruned model	533-425-533	9M(18%)	18%



# Toy Experiment on MNIST

- Model: 784-300-100 fully-connected NN
- FLOPs for a matrix  $W$  is calculated as the size of the smallest sub-matrix formed by such rows and columns that contain all non-zero elements in  $W$ .
- GSP: group sparse prior
- PR: pruning and retraining

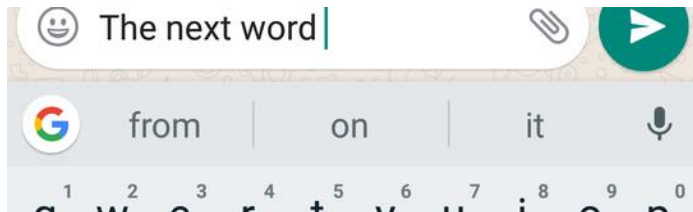
Method	Model	Params	FLOPs	Test Error (%)
SGD (baseline)	1 model	1 <sup>*</sup>	1 <sup>*</sup>	1.66
SGD	18 models	18 $\times$	18 $\times$	1.49
SGLD+GSP+PR	18 models <sup>†</sup>	1.8 $\times$	2.5 $\times$	<b>1.26</b>
SGLD+GSP+PR	18 models <sup>‡</sup>	<b>0.7</b> $\times$	<b>2.2</b> $\times$	1.29

<sup>\*</sup> The baseline model has 266K parameters and 532K FLOPs.

<sup>†</sup> indicates 90% sparsity and <sup>‡</sup> indicates 96% sparsity for each model.

# Language Modeling Experiment

- Language Modeling



- 2-layers LSTM model

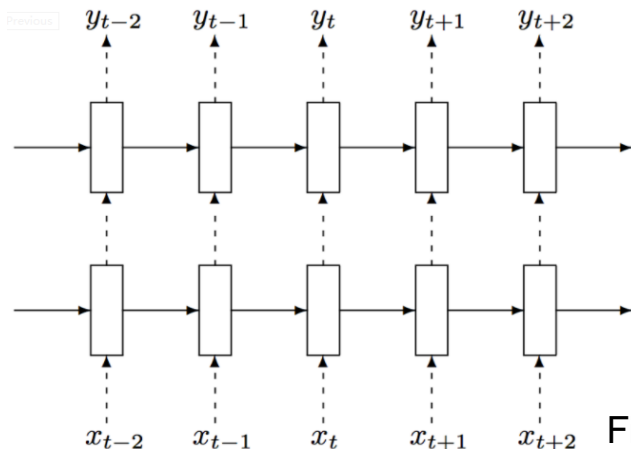


Figure from Zaremba et al. 2014

- Penn Tree Bank dataset

Vocabulary size: 10K

Dataset size: 929K/73K/10K words in training, development and test sets respectively.

- Perplexity

A measurement of how well the language model predicts the word sequence.

$$\text{PPL} = e^{-\frac{1}{N} \sum \log P(w_i)}$$

# Language Modeling Experiment

- Comparison of various models based on LSTMs on PTB dataset.

Method	Model	Params	FLOPs	Dev.	Test
SGD [10]	1 large	1*	1*	82.2	78.4
SGD [10]	38 large	38×	38×	71.9	68.7
VD [24]	10 large	10×	-	-	68.7
VD+SEAL [11]	individual	51M	-	71.1	68.5
SGLD+GSP+PR	20 large	2.0×	4.5×	68.6	66.4
SGLD+GSP+PR	4 large	0.4×	0.5×	72.2	69.7
SGLD+GSP+PR+SE	4 large	<b>0.3×</b>	<b>0.7 ×</b>	<b>64.4</b>	<b>62.1</b>

\* The baseline LSTM model has 66M parameters and 102M FLOPs.

Ref : [10] Wojciech Zaremba et al. 2014; [11] Hakan Inan et al. 2017; [24] Yarin Gal et al. 2016

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# Conclusion & Future Work

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## Conclusion:

- Propose a novel approach for learning ensembles of neural networks.
- Combination of SG-MCMC sampling, group sparse prior and network pruning.
- Experimental verifications for sparse structure learning for LSTM models.

## Future work:

- Interleaving model sampling and model pruning.
- Expand to more tasks.

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**Thank you!**

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