Joint Code Acquisition and Doppler Frequency Shift Estimation for GPS Signals

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***Abstract*—The unavoidable Doppler frequency shift reduces the correlation peak for code acquisition in global positioning system (GPS). In contrast to conventional methods where the code phase and Doppler frequency shift are separately treated, this paper proposes a novel three-step scheme for joint code acquisition and Doppler estimation, whereby not only the Doppler effect on the correlation peak is removed, but also the reduced detection probability due to noise enhancement in low signal-to-noise ratio (SNR) environments is avoided. The theoretical analysis shows that the proposed method has low complexity and fast acquisition speed. Computer simulations demonstrate that code acquisition with high detection probability and Doppler frequency shift estimation with high accuracy can be simultaneously achieved.**

I. Introduction

The acquisition of the global positioning system (GPS) signal poses a two-dimensional search problem of the periodical pseudo-random noise (PN) code, where both the Doppler frequency shift and the phase of the C/A code have to be determined [1]. The basic idea of GPS signal acquisition was to search over a predicted code-frequency uncertainty zone. The sliding correlation with a simple hardware circuit correlated the incoming signal and the local C/A code for a predetermined time interval with a variety of code phases and Doppler frequency candidates [2]. The receiver must search over two dimensional code-frequency planes, thus the acquisition process was time-consuming.

Most GPS receivers use parallel correlations to accelerate the acquisition speed. One could use a bank of correlators employing a straightforward parallel approach or the matched filter (MF), which efficiently calculated the correlations reusing the hardware for different code phases [3]. Acquisition time was shortened through increasing the hardware costs. Alternative technique based on the fast Fourier transform (FFT) in the frequency domain have been extensively studied for fast code acquisition [4]. FFT was used to compute the correlations for all the code phases at one candidate Doppler frequency, so that the average acquisition time was reduced. More recently, some code acquisition algorithms based on the combination of partial match filter (PMF) and FFT have been proposed to further decrease the acquisition time [5]. Those methods used the PMF block for code phase search and the FFT was used for Doppler estimation at the same time. Therefore, the two-dimensional search problem was converted into one-dimensional search, and the acquisition time was minimized.

However, those code acquisition algorithms have some shortcomings: 1) The correlation peak is attenuated due to Doppler frequency shift; 2) Low complexity and fast acquisition speed can not be simultaneously achieved; 3) Limited by either the search step or the FFT size, the frequency search resolution is low. The pre-detection differential (Pre-DD) scheme could suppress the effect of the Doppler shift [6], but the correlation peak was severely attenuated due to the multiplication of the noisy signals with low signal-to-noise ratio (SNR).

To solve those problems of the existing algorithms, this paper proposes a three-step scheme for joint code acquisition with high detection probability and Doppler frequency shift estimation with high accuracy. *Step 1*, by taking both the code phase and Doppler frequency shift into account, differential correlation is used to remove the Doppler effect on the correlation peak. *Step 2*, Doppler shift is estimated using the correlation results derived in the first step with multiple delays. *Step 3*, to avoid the amplitude attenuation and the resulting reduced detection probability in the first step in low SNR conditions, coherent correlation between the Doppler shift compensated incoming signal and the local C/A code is used for the final code acquisition.

The remainder of this paper is organized as follows. Section II illustrates the impact of the Doppler frequency shift on the code acquisition of GPS signals. Section III presents the proposed three-step scheme for joint code acquisition and Doppler shift estimation, including the analysis of the complexity, acquisition time, and Doppler estimation accuracy. The performance of the proposed method is presented in Section IV. We then conclude this paper in Section V.

II. Doppler Shift Impact on GPS Code Acquisition

The received GPS signal $y(n)$ contaminated by multi-path fading, additive white Gaussian noise (AWGN) and Doppler frequency shift can be written as

$$y(n) = \sum_{l=0}^{L-1} h(l)c(n-l)e^{j\Omega n} + w(n), \tag{1}$$

where $c(n)$ is the N-point GPS C/A code with good autocorrelation characteristic, $h(l)$ is the channel impulse response (CIR) modeled as an L-order finite impulse response (FIR)

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filter, Ω is the Doppler frequency shift Δf normalized by the C/A code duration T ($\Omega = 2\pi\Delta f T/N$), and $w(n)$ is the complex Gaussian noise with zero mean and variance of σ^2. The conventional code acquisition methods are based on the correlation between the received signal $y(n)$ and the local C/A code $c(n)$, i.e.,

$$\begin{aligned} R_{yc}(k) &= \sum_{n=0}^{N-1} y(n+k)c^*(n) \\ &= \sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h(l)c(n+k-l)c^*(n)e^{j\Omega(n+k)} + w'(n) \\ &= \begin{cases} h(k)\sum\limits_{n=0}^{N-1}|c(n)|^2 e^{j\Omega(n+k)} + w'(n) & k=l \\ w'(n) & k \neq l \end{cases}, \end{aligned} \tag{2}$$

where $(\cdot)^*$ denotes the complex conjugation, $w'(n)$ is the noise term, and the ideal autocorrelation of the C/A code is assumed. If the first path of the CIR is the strongest, in the presence of Doppler frequency shift Ω, the maximum correlation peak becomes

$$\begin{aligned} R_{yc}(0) &= h(0)\sum_{n=0}^{N-1}|c(n)|^2 e^{j\Omega n} \\ &= h(0)\frac{\sin(\Omega N/2)}{\sin(\Omega/2)} \cdot e^{-j\Omega\frac{N-1}{2}} \\ &= Nh(0)\sin c(\Delta f T)\cdot e^{-j\Omega\frac{N-1}{2}}, \end{aligned} \tag{3}$$

where the noise term $w'(n)$ is ignored for simplicity.

As shown in (3), the negative impacts of the Doppler frequency shift Ω on the correlation output are twofold. Firstly, a constant phase rotation $\Omega(N-1)/2$ is introduced. Secondly, the power of the correlation peak falls off according to a sinc-squared function. In the worst case, the amplitude of the correlation peak $|R_{yc}(0)|$ would be attenuated to zero if $\Omega = 2m/N$, where m is an arbitrary integer except zero. Code acquisition is impossible in this case.

III. Joint Code Acquisition and Doppler Estimation for GPS Signals

In this section, a novel algorithm for joint code acquisition and Doppler estimation is proposed at first. Then the analysis of the computational complexity, the acquisition time, and the Doppler estimation accuracy are presented in the sequel.

A. Joint Code Acquisition and Doppler Estimation

A novel three-step scheme of joint code acquisition and Doppler frequency shift estimation is proposed for GPS signals:

1) *Step 1*: D-lag differential correlation;
2) *Step 2*: Doppler estimation based on multiple delays;
3) *Step 3*: Coherent correlation.

The following parts describe those three steps in detail. In *Step 1*, unlike conventional methods where the correlation is directly applied to the local C/A code and received signal, we use the D-lag differential sequence for both the local C/A code and received signal for correlation.

The D-lag differential sequence of the received signal is presented as

$$\begin{aligned} y_D(n) &= y^*(n)y(n+D) \\ &= \left[c(n)e^{j\Omega n} + w(n)\right]^* \left[c(n+D)e^{j\Omega(n+D)} + w(n+D)\right] \\ &= c^*(n)c(n+D)e^{j\Omega D} + w''(n), \end{aligned} \tag{4}$$

where the channel is assumed to be AWGN, and $w''(n)$ is the noise term. Then the correlation between $y_D(n)$ and corresponding D-lag differential sequence of the local C/A code $c_D(n) = c^*(n)c(n+D)$ yields to the D-lag differential correlation

$$\begin{aligned} R_D(i) &= \sum_{n=0}^{\mathrm{N}-1} c_D^*(n)\cdot y_D(n+i) \\ &\approx e^{j\Omega D}\sum_{n=0}^{\mathrm{N}-1}[c(n)c^*(n+i)][c^*(n+D)c(n+D+i)]. \end{aligned} \tag{5}$$

When the local C/A code is aligned with the received code, the maximum correlation peak is achieved

$$|R_D(0)| = \left| e^{j\Omega D}\sum_{n=0}^{\mathrm{N}-1}|c(n)|^2\cdot|c(n+D)|^2\right| = N. \tag{6}$$

Comparing the correlation peaks in (3) and (6), it is clear that the negative impact of the Doppler frequency shift Ω on the amplitude of the correlation peak has been removed.

However, just like the Pre-DD scheme in [6], the multiplication of the noisy incoming signals with low SNR in (4) would result in severe power loss and consequently the reduced detection probability for code acquisition. This problem can be solved by the following two steps.

After confirming the correlation peak via *Step 1*, the Doppler frequency shift Ω can be directly estimated as

$$\widehat{\Omega} = \frac{\arg\left[R_D(0)\right]}{D}, \tag{7}$$

where $\arg(z)$ is the angle of complex-valued z. The Doppler estimation in (7) is only based on single delay. To improve the accuracy, the Doppler estimation in *Step 2* can be realized by using multiple delays

$$\widehat{\Omega} = \frac{1}{d+1}\sum_{D=D_{\max}-d}^{D_{\max}} \frac{\arg\left[R_D(0)\right]}{D}, \tag{8}$$

where $D_{\max}$ is the maximum of the multiple delays within the range $[D_{\max}-d, D_{\max}]$, d determines the number of delays. If $d=0$, the multiple delays based estimation is simplified to be based on single delay. Just like the direct FFT based acquisition algorithms to reduce the complexity of computing the correlation [4], the whole sequence of $\{R_D(i)\}_{i=0}^{N-1}$ in (5) can be realized by M-point FFT/IFFT ($M = 2N+2$) for higher computational efficiency.

Step 3 correlates the Doppler frequency shift compensated incoming signal $\widehat{y}(n)$ with the local C/A code $c(n)$ for the final code acquisition, whereby the power loss due to the noise enhancement problem can be avoided

$$R_{\widehat{y}c}(0) = \sum_{n=0}^{N-1} \widehat{y}(n)c^*(n) = \sum_{n=0}^{N-1} y(n)e^{-j\widehat{\Omega}n}c^*(n). \quad (9)$$

If the Doppler estimation is accurate enough, i.e., $\widehat{\Omega} = \Omega$, the coherent correlation in (9) would also lead to $R_{\widehat{y}c}(0) = N$.

B. Computational Complexity and Acquisition Time

The sliding correlation based method generates the correlation outputs between the local C/A code holding N different code phases and the received signal, at the end of every N chips [2]. So its computational complexity in terms of times of multiplication is $O_1 = N^2$. In addition, since the correlation is not immune to the Doppler frequency shift, the Doppler search is required in the range of Doppler frequency uncertainty $[-f_{\max}, +f_{\max}]$, with the serial Doppler search step of f_d. Therefore, the maximum acquisition time over the two-dimensional code-frequency planes is $T_1 \triangleq T_{\max} = (2f_{\max}/f_d)NT = MNT$, where $M = 2f_{\max}/f_d$ is the number of Doppler frequency trial.

The MF based correlation can produce the correlation output at the speed of every chip with the increase of hardware complexity due to the adoption of high-order MF [3]. It also needs N^2 times of multiplication but the acquisition time is shortened by N times. So its complexity is $O_2 = O_1 = N^2$ and the acquisition time is $T_2 = T_1/N = MT$.

The direct FFT based method [4] makes use of the complexity-saving algorithm of M-point FFT/IFFT to realize the calculation of correlation, where $M = 2N + 2$. However, the Doppler search is still required. Therefore, its complexity is $O_3 = 3 \times (M/2)\log_2 M + M = 3(N+1)\log_2(N+1) + 5(N+1)$, and the acquisition time is $T_3 = T_2 = MT$.

It is found that the correlation peaks of all the above three types of methods are greatly influenced by the Doppler shift, leading to the fact that the serial search for Doppler frequency shift is indispensable for those methods. However, the PMF-FFT based algorithms [5] apply FFT to the partial correlation outputs coming from PMFs to alleviate the impact of Doppler frequency shift on the correlation peak, and the coarse frequency estimation can be simultaneously achieved by selecting the maximum output of FFT, so that the serial Doppler search is avoided. If the correlation is divided into P partial correlations implemented by P parallel PMFs, and then P-point FFT is applied, its acquisition complexity would be $O_4 = [(P/2)\log_2 P + P(N/P)]N = N(P/2)\log_2 P + N^2$. Its acquisition time is reduced by M times due to the avoidance of serial Doppler shift search. That is $T_4 = T_3/M = T$.

For the proposed joint code acquisition and Doppler estimation algorithm in this paper, the complexity of the FFT process for single delay based correlation is the same as that for the direct FFT based method. Therefore, its complexity is $O_5 = 3N + O_3 = 3(N+1)\log_2(N+1) + 8N + 5$. Its acquisition time is the same as that of the PMF-FFT based methods, since the Doppler estimation in (8) can be directly achieved from the FFT outputs, So $T_5 = T_4 = T$.

TABLE I
PERFORMANCE COMPARISON OF THE PROPOSED ALGORITHM AND OTHER FOUR TYPES OF THE STATE-OF-THE-ART METHODS.

Algorithm	Computational Complexity	Acquisition Time
Sliding correlation	N^2	MNT
MF based	N^2	MT
Direct FFT based	$3(N+1)\log_2(N+1) + 5(N+1)$	MT
PMF-FFT based	$N(P/2)\log_2 P + N^2$	T
Proposed	$3(N+1)\log_2(N+1) + 8N + 5$	T

In summary, the comparison of the computational complexity and acquisition time between the proposed method and other four types of existing methods is listed in Table I.

It is clear that for the four types of existing algorithms, the direct FFT based one has the lowest complexity and the PMF-FFT based one has the fastest acquisition speed. However, low complexity and fast acquisition speed can not be simultaneously achieved. On the other hand, the proposed method has the smallest acquisition time of T, while its complexity is only slightly higher than the direct FFT based method. Therefore, low complexity and fast acquisition speed could be simultaneously achieved by the proposed three-step scheme.

C. Doppler Estimation Accuracy

The Doppler estimation accuracy is limited by the serial Doppler search step f_d for the three types of traditional algorithms based on sliding correlation, MF and direct FFT, because the correlation peaks of those three methods are all attenuated by the Doppler shift. The typical serial search step of the Doppler frequency shift is configured as 500 Hz.

As for the direct FFT based algorithm, the Doppler estimation can also be achieved in the frequency domain after code phase is aligned, according to the following time-frequency property of FFT

$$c(n)e^{j\Omega n} = c(n)e^{j\frac{2\pi}{N}n\cdot(\Omega\frac{N}{2\pi})} = IFFT\left[X((k - \Omega\frac{N}{2\pi}))_N\right], \quad (10)$$

where $X(k) = FFT\,[c(n)]$, and $((*))_N$ means the cyclic shift with the period of N samples. $\Omega(N/2\pi)$ in (10) must be an integer for Doppler estimation, so the frequency resolution is $\Delta f = 1/T$. For GPS signal, Δf equals to 1 kHz.

The Doppler estimation resolution for PMF-FFT based algorithm also equals to $1/T$. On the other hand, the resolution can be improved by interpolation of the FFT output in the frequency domain to several hundreds of Hertz [5].

The proposed Doppler estimation based on multiple delays in (8) can be interpreted as an averaged version of the Kay estimator [7]. The theoretical variance of the estimates in the moderately high SNR regions has been derived as [8]

$$\text{var}(\widehat{\Omega}) = \frac{1}{\gamma D_{\max}(N - D_{\max})^2}, \quad (11)$$

where $\gamma \triangleq 1/2\sigma^2$ is the SNR.

The Doppler estimation accuracy of the proposed method can be evaluated by the root mean square error (RMSE) of the Doppler estimates, which is defined as

$$RMSE(\Omega) = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left(\widehat{\Omega}_k - \Omega\right)^2}, \quad (12)$$

where $\widehat{\Omega}_k$ is the estimates of the normalized Doppler frequency shift Ω, and K is the total number of the observation of $\widehat{\Omega}_k$.

The computer simulation in Section IV will demonstrate that the Doppler estimation accuracy of the proposed algorithm is the best among those five types of methods mentioned above.

D. The Selection of Delay

The theoretical estimation accuracy of the proposed scheme expressed in (11) indicates that the RMSE of the Doppler estimation is neither decided by the serial search step nor the FFT size, but is inversely proportional to the delay $D_{\max}$ if $D_{\max} < N/3$. However, the phase unwrapping in (8) determines the Doppler estimation range of the proposed algorithm, which reads

$$\Delta f \in \left(-\frac{N}{2TD_{\max}}, \frac{N}{2TD_{\max}}\right]. \quad (13)$$

The Doppler frequency uncertainty region for GPS receiver without almanac information is $[-f_{\max}, f_{\max}]$, where $f_{\max}$ is typically 10 kHz [9], so the maximum possible candidate delay $D_{\max}$ should be

$$D_{\max} = \frac{N}{2Tf_{\max}}. \quad (14)$$

We select $D_{\max} = 50$ for general GPS receiver to reserve a little margin for the Doppler estimation range. This number can be configured according to the practical requirements of Doppler estimation accuracy and computational complexity.

In addition, the estimation range can be maximized to be $(-N/2T, N/2T]$ if we set $D_{\max} = 1$ and $d = 0$ in (8), or the differential correlation between adjacent elements of $\{R_D(0)\}_{D=D_{\max}-d}^{D_{\max}}$ is applied for Doppler estimation. However, large d leads to high computational complexity.

IV. Simulation Results and Discussions

Simulations are carried out to evaluate the performance of the proposed joint code acquisition and Doppler estimation scheme. The main simulation parameters are configured as: 1) The C/A code with the length of $N = 1023$; 2) The chip rate of the C/A code is $R_c = 1.023$ Mc/s ; 3) Both AWGN channel and two ITU-R defined multi-path channels called Vehicular A and Vehicular B [10] are used for simulation.

Fig. 1 compares the detection probability of the proposed algorithm with the PMF-FFT [5] and Pre-DD [6] based methods under AWGN channel with the Doppler shift of 1 kHz. The detection threshold is configured to guarantee the false alarm probability (FAP) is smaller than 10^{-3}. It is shown that the detection probability of the proposed algorithm approaches 100% when SNR is above -5 dB, while it remains constant 0% for the PMF-FFT based method over the whole SNR range for simulation. The reason is that the correlation peak of the PMF-FFT based method is greatly attenuated due to the presence of Doppler shift, which has no impact on the proposed algorithm for relatively high SNR. Compared with the Pre-DD algorithm, obvious SNR gain can be achieved by the proposed scheme, where the power loss due to multiplication of the noisy signal with low SNR is avoided by the Doppler shift compensated incoming signal in *Step 3*.

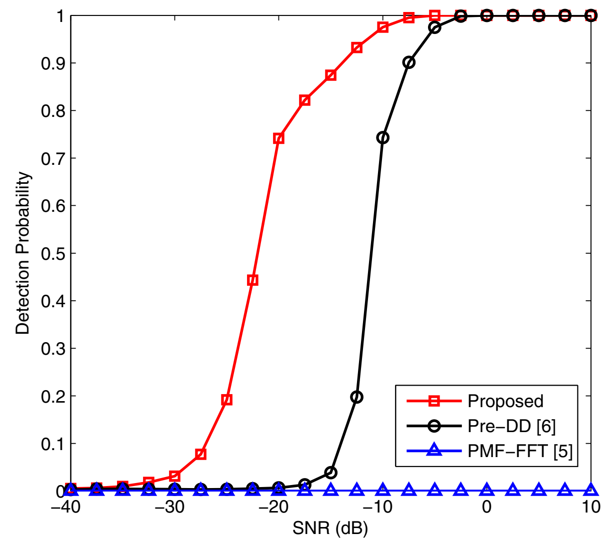

Fig. 1. Detection probability under AWGN channel with the Doppler frequency shift of 1 kHz.

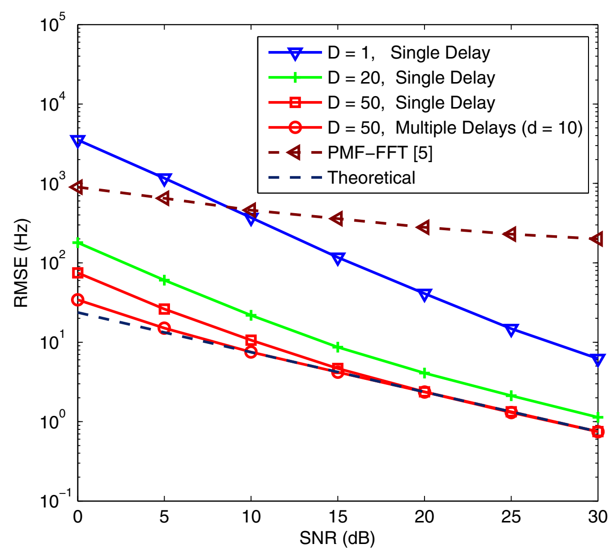

Fig. 2. Estimation accuracy of the Doppler frequency shift under AWGN channel when different delays are applied for the proposed algorithm.

Fig. 2 compares the estimation accuracy of the Doppler frequency shift of 2.5 kHz under AWGN channel when different

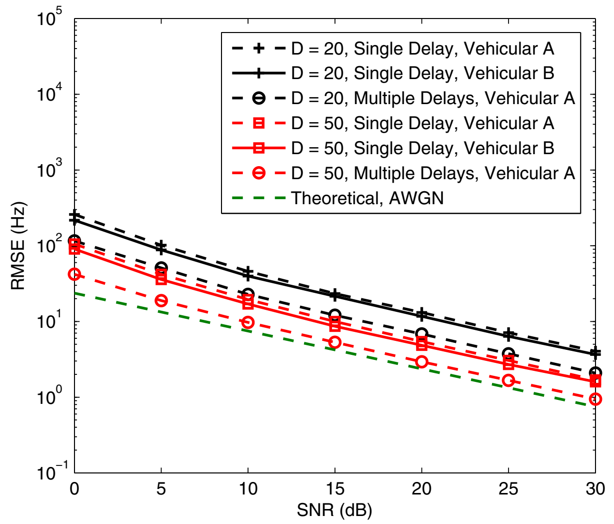

Fig. 3. Estimation accuracy of the Doppler frequency shift under multi-path channels when different delays are applied for the proposed algorithm.

delays are considered. The estimation uncertainty of the PMF-FFT based algorithm adopting interpolation [5] is presented for comparison, and the theoretical RMSE directly obtained from (11) is also included for reference. We can find that when the single delay is selected as 50, the RMSE of the Doppler estimation is in the range of [0.8 Hz, 80 Hz] under the SNR range of [0 dB, 30 dB], while the estimation uncertainty of the PMF-FFT based algorithm is in the range of [200 Hz, 900 Hz]. It also shows that the simulated RMSE almost reaches the theoretical RMSE when SNR is above 15 dB. Multiple delays based Doppler estimation with higher complexity can improve the accuracy, especially in low SNR regions.

Fig. 3 shows the estimation accuracy of the Doppler frequency shift of 2.5 kHz under two typical multi-path channels. The simulation results show that the RMSE deteriorates a little in multi-path environments, and the Doppler shift estimation accuracy is in the order of tens of Hertz. Compared with the estimation accuracy under AWGN channel, multiple delays based Doppler estimation can achieve higher performance gain under multi-path channels.

V. Conclusion

A three-step scheme of joint code acquisition and Doppler frequency shift estimation with good performance for GPS signals is proposed in this paper. Taking the Doppler frequency shift into consideration during the correlation calculation, the peak attenuation effect is eliminated by the delay-multiplication operation of both the local C/A code and received signal before FFT is applied to compute the correlation. Then the Doppler frequency shift is derived with high accuracy based on single or multiple delays. Finally, the coherent correlation using the Doppler shift compensated incoming signal can avoid the problem of noise enhancement in other differential algorithms. Theoretical analysis shows that the proposed scheme has the fastest acquisition speed just like the PMF-FFT based algorithm, and it has low complexity similar to the direct FFT based method. High detection probability and accurate Doppler estimation can be simultaneously achieved. The proposed scheme is also applicable to timing synchronization and spectrum sensing in wireless scenarios like CDMA systems or PN training sequence based OFDM systems.

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References

[1] E. D. Kaplan and C. Hegarty, *Understanding GPS: Principles and Applications*, 2nd ed. Boston, USA: Artech House Publishers, 2005.

[2] J. C. Lin, "Noncoherent sequential PN code acquisition using sliding correlation for chip-asynchronous direct-sequence spread-spectrum communications," *IEEE Trans. Commun.*, vol. 50, no. 4, pp. 664–676, Apr. 2002.

[3] A. Polydoros and C. Weber, "A unified approach to serial search spread-spectrum code acquisition–part II: A matched-filter receiver," *IEEE Trans. Commun.*, vol. 32, no. 5, pp. 550–560, May 1984.

[4] P. K. Sagiraju, G. V. S. Raju, and D. Akopian, "Fast acquisition implementation for high sensitivity global positioning systems receivers based on joint and reduced space search," *IET Radar Sonar and Navigation*, vol. 2, no. 5, pp. 376–387, Oct. 2008.

[5] S. M. Spangenberg, I. Scott, and S. McLaughlin, "An FFT-based approach for fast acquisition in spread spectrum communication systems," *Wireless Personal Communications*, vol. 13, no. 1-2, pp. 27–56, May 2000.

[6] S. K. Shanmugam, R. Watson, J. Nielsen, and G. Lachapelle, "Differential signal processing schemes for enhanced GPS acquisition," in *Proc. 2005 International Symposium on IEEE ION/GNSS*, Sept. 2005, pp. 212–222.

[7] S. Kay, "A fast and accurate single frequency estimator," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, no. 12, pp. 1987–1990, Dec. 1989.

[8] P. Handel, A. Eriksson, and T. Wigren, "Performance analysis of a correlation based single tone frequency estimator," *IEEE Trans. Signal Processing*, vol. 44, no. 2, pp. 223–231, June 1995.

[9] H. Seo, C. Park, and S. Lee, "A new fast acquisition algorithm for GPS receivers," in *Proc. 2000 International Symposium on IEEE GPS/GNSS*, Dec. 2000.

[10] *Guideline for Evaluation of Radio Transmission Technology for IMT-2000*. Recommendation ITU-R M.1225, 1997.