Low-Complexity MMSE Signal Detection Based on Richardson Method for Large-Scale MIMO Systems

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***Abstract*—Minimum mean square error (MMSE) signal detection is near-optimal for uplink multi-user large-scale MIMO systems with hundreds of antennas at the base station, but involves matrix inversion with high complexity. In this paper, we first prove that the filtering matrix of the MMSE algorithm in large-scale MIMO is symmetric positive definite, based on which we propose a low-complexity signal detection algorithm by exploiting the Richardson method to avoid the complicated matrix inversion. The proof of the convergence of the proposed scheme is also provided. We then propose a zone-based initial solution by simply checking the values of the received signals, which can accelerate the convergence rate of the Richardson method for high-order modulations to reduce the complexity further. The analysis shows that the complexity can be reduced from $\mathcal{O}(K^3)$ to $\mathcal{O}(K^2)$ by the proposed signal detection algorithm, where K is the number of users. Simulation results indicate that the proposed algorithm outperforms the recently proposed Neumann series approximation algorithm and achieves the near-optimal performance of the classical MMSE algorithm.**

I. Introduction

Multiple-input multiple-output (MIMO) technology has been successfully integrated in various wireless communication systems like the 4th generation (4G) cellular system LTE-A, and IEEE 802.11n wireless LAN system. It is envisaged to be a key feature of future advanced wireless systems [1]. Unlike the conventional MIMO with small number of antennas (e.g., at most 4 for uplink in LTE-A), large-scale MIMO, where the base station (BS) is equipped with a very large number of antennas (e.g., 128 antennas or even more) to simultaneously serve multiple user equipments (UEs), is recently proposed [2]. It has been proved that large-scale MIMO provides potential opportunity to achieve orders of magnitude increase in spectral and energy efficiency [3], [4].

Building a practical system to realize the very attractive merits of large-scale MIMO is not a trivial task, and one challenging problem is the low-complexity signal detection algorithm in the uplink [5]. The optimal detector is the maximum likelihood (ML) detector, but its complexity increases exponentially with the number of transmit antennas, making it impractical for large-scale MIMO. The fixed-complexity sphere decoding (FSD) [6] and tabu search (TS) [7] algorithms are proposed to achieve near optimal performance with reduced complexity. However, their complexity is unfordable when the dimension of MIMO system is large or the modulation order is high [8] (e.g., 128 antennas at the BS with 64 QAM modulation). One has to resort to low-complexity linear detection algorithms such as zero-forcing (ZF) and minimum mean square error (MMSE) which are near-optimal for uplink multi-user large-scale MIMO systems [5], but such algorithms involve unfavorable matrix inversion, whose complexity is still high for large-scale MIMO. Recently, to simplify the matrix inversion, [9] proposed Neumann series approximation algorithm, which can convert the matrix inversion into a series of matrix-vector multiplications. However, only marginal reduction in complexity can be achieved.

In this paper, we propose a low-complexity near-optimal signal detection algorithm to avoid the complicated matrix inversion. We first prove a special property of large-scale MIMO systems that the MMSE filtering matrix is symmetric positive definite, based on which we propose to exploit the Richardson method [10] to avoid the complicated matrix inversion. Then we prove that the proposed algorithm is sure to converge for any initial solution when the relaxation parameter is appropriate. To further accelerate the convergence rate and reduce the computational complexity, we propose a zone-based initial solution to the Richardson method, which enjoys a fast convergence rate and consequently reduces the required number of iterations. We verify through simulations that the proposed signal detection algorithm can efficiently solve the matrix inversion problem in an iterative way until the desired accuracy is attained. Alternatively, for a given affordable delay, our algorithm is able to provide a better accuracy for a limited number of iteration.

The rest of the paper is organized as follows. Section II describes the system model. The proposed low-complexity signal detection algorithm, the method to find the initial solution, and the complexity analysis are provided in Section III. Section IV shows the simulation results of the bit error rate (BER) performance. Finally, conclusions are drawn in Section V.

Notation: We use lower-case and upper-case boldface letters to denote vectors and matrices, respectively; $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, and $|\cdot|$ denote the transpose, conjugate transpose, matrix inversion, and absolute operators, respectively; $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real part and imaginary part of a complex number, respectively; $\mathbf{1}_N$ denote the $N \times 1$ vector whose entries are all set as one; Finally, $\mathbf{I}_N$ is the $N \times N$ identity matrix.

II. System Model

Consider a uplink large-scale multi-user MIMO system which employs N antennas at the BS to simultaneously

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serve K single-antenna UEs [2], [3], [5]. For large-scale MIMO, we usually have $N >> K$, e.g., $N = 128$ and $K = 16$ was considered in [5]. The encoded bits are modulated by taking symbols from a set of constellation alphabet Q, e.g., quadrature amplitude modulation (QAM). Let $\mathbf{s}_c \in Q$ denote the $K \times 1$ transmitted signal vector of complex values and $\mathbf{H}_c \in \mathbb{C}^{N \times K}$ denote the flat Rayleigh fading channel matrix, whose entries are assume to be independently and identically distributed (i.i.d.) with zero mean and unit variance [3]. Then the $N \times 1$ received signal vector $\mathbf{y}_c$ at the BS can be represented by

$$\mathbf{y}_c=\mathbf{H}_c\mathbf{s}_c+\mathbf{n}_c, \tag{1}$$

where $\mathbf{n}_c$ is a $N \times 1$ zero-mean additive white Gaussian noise (AWGN) vector each entry of which has the variance σ^2.

For signal detection, the complex-valued system model (1) can be also directly converted to a corresponding real-valued system model

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{2}$$

where $\mathbf{y} = [\mathrm{Re}\{\mathbf{y}_c\} \quad \mathrm{Im}\{\mathbf{y}_c\}]^T$ is the $2N \times 1$ real-valued received signal vector, accordingly $\mathbf{s} = [\mathrm{Re}\{\mathbf{s}_c\} \quad \mathrm{Im}\{\mathbf{s}_c\}]^T$, $\mathbf{n} = [\mathrm{Re}\{\mathbf{n}_c\} \quad \mathrm{Im}\{\mathbf{n}_c\}]^T$, and

$$\mathbf{H} = \begin{bmatrix} \mathrm{Re}\{\mathbf{H}_c\} & -\mathrm{Im}\{\mathbf{H}_c\} \\ \mathrm{Im}\{\mathbf{H}_c\} & \mathrm{Re}\{\mathbf{H}_c\} \end{bmatrix}_{2N \times 2K}. \tag{3}$$

The task of multi-user signal detection at the BS is to estimate the transmitted signal vector $\mathbf{s}$ from the received signal vector $\mathbf{y}$. Note that the channel matrix $\mathbf{H}$ can be usually achieved by time-domain and/or frequency-domain training pilots [11] [12]. It has been proved that minimum mean-square error (MMSE) linear detection algorithm is near-optimal for uplink multi-user large-scale MIMO systems [5], and the estimate of the transmitted signal vector $\hat{\mathbf{s}}$ coming from K different users can be achieved by

$$\hat{\mathbf{s}} = (\mathbf{H}^H\mathbf{H} + \sigma^2\mathbf{I}_{2K})^{-1}\mathbf{H}^H\mathbf{y} = \mathbf{W}^{-1}\hat{\mathbf{y}}, \tag{4}$$

where

$$\hat{\mathbf{y}} = \mathbf{H}^H\mathbf{y}, \tag{5}$$

can be interpreted as the matched-filter output of $\mathbf{y}$, and the MMSE filtering matrix $\mathbf{W}$ is denoted by

$$\mathbf{W} = \mathbf{G} + \sigma^2\mathbf{I}_{2K}, \tag{6}$$

where $\mathbf{G} = \mathbf{H}^H\mathbf{H}$ denotes the Gram matrix. Note that the direct computation of the matrix inversion $\mathbf{W}^{-1}$ requires high complexity of $\mathcal{O}(K^3)$.

III. Low-Complexity Signal Detection For Uplink Large-Scale MIMO

In this section, we first prove that the MMSE filtering matrix in large-scale MIMO systems is symmetric positive definite, based on which we propose a low-complexity signal detection algorithm using Richardson method without matrix inversion, together with the convergence proof. We then propose a zone-based initial solution to the Richardson method, which can accelerate the convergence rate for high-order modulations. Finally, we provide the complexity analysis of the proposed algorithm.

A. Signal detection based on Richardson method

Lemma 1. *For signal detection in uplink large-scale MIMO systems, the MMSE filtering matrix* $\mathbf{W}$ *is symmetric positive definite.*

Proof: For uplink large-scale MIMO systems where $N \gg K$, the real-valued channel matrix $\mathbf{H}$ has full column rank [5] (e.g., $\mathrm{rank}\,(\mathbf{H}) = 2K$). Then the equation $\mathbf{H}\mathbf{q} = 0$ has an unique solution, i.e., $\mathbf{q}$ is the $2K \times 1$ zero vector. Thus, for any arbitrary $2K \times 1$ non-zero vector $\mathbf{r}$, we have

$$(\mathbf{H}\mathbf{r})^H\mathbf{H}\mathbf{r} = \mathbf{r}^H(\mathbf{H}^H\mathbf{H})\mathbf{r} = \mathbf{r}^H\mathbf{G}\mathbf{r} > 0, \tag{7}$$

which indicates the Gram matrix $\mathbf{G}$ $=\mathbf{H}^H\mathbf{H}$ is positive definite. In addition, as we have

$$\mathbf{G}^H = (\mathbf{H}^H\mathbf{H})^H = \mathbf{G}, \tag{8}$$

so $\mathbf{G}$ is symmetric. Therefore, the Gram matrix $\mathbf{G}$ is symmetric positive definite.

Finally, as the noise variance σ^2 is positive, we can conclude that the MMSE filtering matrix $\mathbf{W} = \mathbf{G} + \sigma^2\mathbf{I}_{2K}$ is symmetric positive definite, too. ■

The special property that the MMSE filtering matrix $\mathbf{W}$ in uplink large-scale MIMO systems is symmetric positive definite inspires us to exploit the Richardson method to efficiently solve (4) in an inversion-less way. The Richardson method is used to solve N-dimension linear equation formulated as

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{9}$$

where $\mathbf{A}$ is the symmetric positive definite matrix, $\mathbf{x}$ is the $N \times 1$ solution vector, and $\mathbf{b}$ is the $N \times 1$ measurement vector. The Richardson iteration can be described as [10]

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + w(\mathbf{b} - \mathbf{A}\mathbf{x}^{(i)}), \quad i = 0, 1, 2, \cdots \tag{10}$$

where the superscript i denotes the number of iterations, and w represents the relaxation parameter.

Since $\mathbf{W}$ is symmetric positive definite as proved above, we can exploit the Richardson method to estimate the transmitted signal vector $\hat{\mathbf{s}}$ without matrix inversion as below

$$\mathbf{s}^{(i+1)} = \mathbf{s}^{(i)} + w(\hat{\mathbf{y}} - \mathbf{W}\mathbf{s}^{(i)}), \quad i = 0, 1, 2, \cdots \tag{11}$$

where $\mathbf{s}^{(0)}$ denotes the initial solution, which will be addressed in detail later in Section III-B. The relaxation parameter w in (10) plays an important role in convergence of the Richardson method, and next we will prove that the Richardson method is sure to converge for any initial solution when the relaxation parameter w is appropriately set.

Lemma 2. *For the* N*-dimension linear equation* $\mathbf{A}\mathbf{x} = \mathbf{b}$*, the necessary and sufficient conditions for convergence of Richardson method is that the relaxation parameter satisfies*

$0 < w < 2/\lambda_1$, *where* λ_1 *is the largest eigenvalue of the symmetric positive definite matrix* $\mathbf{A}$.

Proof: We define $\mathbf{D} = \mathbf{I}_N - w\mathbf{A}$ and $\mathbf{c} = w\mathbf{b}$, where $\mathbf{D}$ is the iteration matrix. Then the Richardson iteration (10) can be rewritten as

$$\mathbf{x}^{(i+1)} = \mathbf{D}\mathbf{x}^{(i)} + \mathbf{c}, \quad i = 0, 1, 2, \cdots \tag{12}$$

The spectral radius of iteration matrix $\mathbf{D}$ is the nonnegative number $\rho(\mathbf{D}) = \max_{1 \le n \le N} |\mu_n(\mathbf{D})|$, where $\mu_n(\mathbf{D})$ denotes the nth eigenvalue of $\mathbf{D}$, and the necessary and sufficient conditions for the convergence of (12) is that the spectral radius should satisfies [9, Theorem 7.2.2]

$$\rho(\mathbf{D}) = \max_{1 \le n \le N} |\mu_n(\mathbf{D})| < 1. \tag{13}$$

For simplicity but without loss of generality, we use $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_N > 0$ to denote the N eigenvalues of the symmetric positive definite matrix $\mathbf{A}$, where λ_1 is the largest one. Because of $\mathbf{D} = \mathbf{I}_N - w\mathbf{A}$, we have

$$\mu_n(\mathbf{D}) = 1 - w\lambda_n, \tag{14}$$

where λ_n denotes the nth eigenvalue of $\mathbf{A}$. Substituting (14) into (13), we have

$$0 < w < 2/\lambda_1. \tag{15}$$

■

It is clear from (15) that when the relaxation parameter w is appropriately set after the channel matrix $\mathbf{H}$ has been obtained (thus $\mathbf{W}$ in (4), or equivalently $\mathbf{A}$ above) through frequency-domain and/or time-domain training pilots in large-scale MIMO systems [11] [12], the convergence of Richardson method can be guaranteed.

B. Zone-based initial solution

The last task in the Richardson method is to determine the initial solution, which is usually set as a zero vector [10]. Although the initial solution doesn't influence the convergence, it plays an important role in the convergence rate and affects both computational complexity and estimation performance when the number of iterations is limited. In this part, we propose a zone-based initial solution with low complexity, which can be used for high order constellations.

Lemma 3. *For uplink large-scale MIMO systems, we always have* $\hat{s}_i\hat{y}_i > 0$*, where* $\hat{s}_i$ *and* $\hat{y}_i$ *denote the* i*th element of* $\hat{\mathbf{s}}$ *and* $\hat{\mathbf{y}}$*, respectively.*

Proof: Let ${W_{ij}}^{-1}$ denote the ith row and jth column element of $\mathbf{W}^{-1}$, then according to (4) we have

$$\hat{s}_i\hat{y}_i = \left(\sum_{j=1}^{2K} W_{ij}^{-1}\hat{y}_j\right)\hat{y}_i. \tag{16}$$

Based on the fact that the MMSE filtering matrix $\mathbf{W}$ in large-scale MIMO systems is diagonally dominant [5], $\mathbf{W}^{-1}$ is also diagonally dominant. Additionally, as we have proved that $\mathbf{W}$ is symmetric positive definite in **Lemma 1**, the diagonal entries of $\mathbf{W}^{-1}$ are all positive. Thus, (16) can be approximated as

$$\hat{s}_i\hat{y}_i = \left(\sum_{j=1}^{2K} W_{ij}^{-1}\hat{y}_j\right)\hat{y}_i \approx W_{ii}^{-1}\hat{y}_i\hat{y}_i > 0. \tag{17}$$

■

Input: 1) The number of non-overlapped zones Z;
2) The cardinality of the real constellation $|Q|$;
3) $\hat{y}_i$, the ith element of matched-filter output $\hat{\mathbf{y}}$ in (5).
Output: ${s_i}^{(0)}$, the ith element of initial solution $\mathbf{s}^{(0)}$.
If $\hat{y}_i > 0$
1) Calculate $Z/2 - 1$ boundary values $z = 2n\,|Q|\,/Z, n = 1, 2, \cdots, Z/2 - 1$;
2) Use (18) to decide which zone $\hat{s}_i$ belongs to, e.g., $\hat{s}_i \subset [2n\,|Q|\,/Z, 2(n+1)\,|Q|\,/Z]$;
3) Locate ${s_i}^{(0)}$ at the center of this specific zone.
else if $\hat{y}_i < 0$
1) Calculate $Z/2 - 1$ boundary values $z = -2n\,|Q|\,/Z, n = 1, 2, \cdots, Z/2 - 1$;
2) Use (18) to decide which zone $\hat{s}_i$ belongs to, e.g., $\hat{s}_i \subset [-2(n+1)\,|Q|\,/Z, -2n\,|Q|\,/Z]$;
3) Locate ${s_i}^{(0)}$ at the center of this specific zone.
else
${s_i}^{(0)} = 0$.
end

Algorithm 1: Zone-based initial solution

Lemma 3 implies that we can simply determine the sign of $\hat{s}_i$ according to $\hat{y}_i$. Based on that, we can further narrow the potential range of the initial solution $\mathbf{s}^{(0)}$ in (11). For example, we can calculate

$$\tilde{\mathbf{y}} = \hat{\mathbf{y}} - \mathbf{W} \times (\underbrace{z, z, \cdots, z}_{2K})^T, \tag{18}$$

where z is a constant integer, and then we can decide whether $\hat{s}_i$ is larger than z or not according to $\tilde{y}_i$ (the ith element of $\tilde{\mathbf{y}}$). This inspires us come to the idea of zone-based initial solution by selecting several different z's to divide the potential range of the initial solution into multiple non-overlapped zones, then we can determine the initial solution $\mathbf{s}^{(0)}$ belongs to which specific zone according to (18), and consequently fast convergence can be expected.

The proposed zone-based initial solution is described in **Algorithm 1**, and is also illustrated in Fig. 1, where 64 QAM and $Z = 4$ is taken as an example. As s_i can be $-7, -5, -3, -1, +1, +3, +5$, or $+7$, the cardinality of the real constellation is $|Q| = 8$. We first determine which quadrant $\hat{s}_{c,i} = \hat{s}_i + j\hat{s}_{i+K}$ belongs to by simply checking the sign of $\hat{y}_i$ and $\hat{y}_{i+K}$, then we can divide the whole possible values into $Z = 4$ zones to determine the initial solution, so the boundary values for different zones are $z = -4, +4$. Then, when $\hat{y}_i > 0$, we have ${s_i}^{(0)} = +6$ if $\hat{s}_i > +4$, and otherwise ${s_i}^{(0)} = +2$; When $\hat{y}_i < 0$, we have ${s_i}^{(0)} = -6$ if $\hat{s}_i < -4$, and otherwise ${s_i}^{(0)} = -2$.

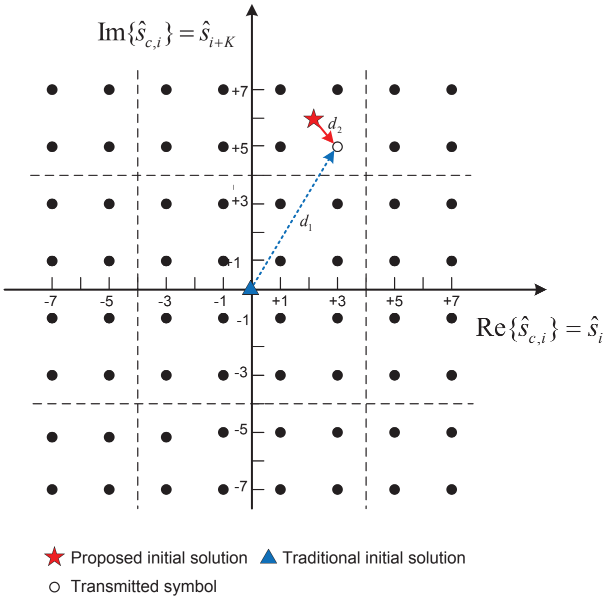

Fig. 1. Illustration of the zone-based initial solution to the Richardson method, where 64 QAM and $Z = 4$ are considered. $\hat{s}_{c,i}$ represents the ith element of the complex-valued estimate of transmitted signal vector $\hat{\mathbf{s}}_c$.

We can observe from Fig. 1 that the distance between the conventional zero vector initial solution and the transmitted symbol is d_1, while the distance between the zone-based initial solution and the transmitted symbol is d_2, which is obviously shorter than d_1. The proposed zone-based initial solution provides a freedom of choice for the appropriate initial solution close to the final solution $\hat{\mathbf{s}}$, which is crucial to ensure a fast convergence rate of the Richardson method, especially for high-order constellations like 64 QAM and 256 QAM. Note that although the regular square QAM is considered in this paper, the mechanism of the proposed zone-based initial solution can be easily extended to other modulations like APSK and non-uniform constellations [13].

C. Computational complexity analysis

The computational complexity in terms of number of multiplications, required by the computation of $\mathbf{W}^{-1}$, is analyzed in this part for clarity. The complexity of the proposed algorithm comes from two parts: 1) The Richardson method; 2) The zone-based initial solution.

TABLE I
COMPUTATIONAL COMPLEXITY

	Conventional Neumann series approximation algorithm [9]	Proposed signal detection algorithm with zone-based initial solution
$i = 2$	$8K^2 - 2K$	$8K^2 + 6K$
$i = 3$	$8K^3$	$12K^2 + 8K$
$i = 4$	$16K^3 - 4K^2$	$16K^2 + 10K$
$i = 5$	$24K^3 - 8K^2$	$20K^2 + 12K$

It can be found from (11) that the ith iteration of the Richardson method involves one multiplication of a $2K \times 2K$ matrix $\mathbf{W}$ and a $2K \times 1$ vector $\mathbf{s}^{(i)}$, as well as one multiplication of a constant relaxation parameter w and a $2K \times 1$ vector $\hat{\mathbf{y}} - \mathbf{W}\mathbf{s}^{(i)}$, thus $4K^2 + 2K$ times of multiplications are required for each iteration. From (18), one can find that the zone-based initial solution only requires $Z/2 - 1$ (where Z is the number of non-overlapped zones) times of multiplications of a $2K \times 2K$ matrix $\mathbf{W}$ and a $2K \times 1$ vector, where the $2K \times 1$ vector is a multiplication of a constant and $\mathbf{1}_{2N}$. When $Z = 4$ is selected for 64 QAM as shown in Fig. 1, the required number of multiplications is as small as $2K$.

Table I compares the complexity of the conventional Neumann series approximation algorithm [9] and the proposed algorithm based on Richardson method for signal detection. It is well known that the complexity of the classical MMSE algorithm is $\mathcal{O}(K^3)$, and Table I shows that the conventional Neumann series approximation algorithm can reduce the complexity from $\mathcal{O}(K^3)$ to $\mathcal{O}(K^2)$ when the number of iterations is $i = 2$, but its complexity is still $\mathcal{O}(K^3)$ when $i \geq 3$. Since usually large value of i is required to ensure the final approximation performance (e.g., $i = 5$ as will be verified later by simulation results in Section IV), the overall complexity is still $\mathcal{O}(K^3)$, which means only marginal complexity reduction can be achieved. However, the complexity of the proposed algorithm is considerably reduced from $\mathcal{O}(K^3)$ to $\mathcal{O}(K^2)$ for any arbitrary number of iterations. Additionally, as the proposed zone-based initial solution is close to the actual solution, the Richardson method can converge fast to the final estimate. Thus, the required number of iterations to achieve a certain estimation accuracy becomes smaller, which means the complexity of the proposed algorithm can be reduced further.

IV. SIMULATION RESULTS

The simulation results of BER performance are provided to compare the proposed signal detection algorithm with the recently proposed Neumann series approximation algorithm in [9]. The BER performance of the classical MMSE algorithm with complicated but exact matrix inversion is also included as the benchmark for comparison. We consider a $N \times K = 128 \times 16$ large-scale MIMO system employing the modulation scheme of 64 QAM, and the rate-1/2 convolutional code with $[133_o\ 171_o]$ polynomial together with a random interleaver. We adopt Rayleigh fading channel model. At the receiver, log-likelihood ratios (LLRs) are extracted for soft-input Viterbi decoding [14]. For simplicity but without loss of generality, we set $Z = 4$ for 64 QAM to select the initial solution to the Richardson method. Through intensive simulations, we find out that when N and K are fixed, the largest eigenvalue of the MMSE filtering matrix $\mathbf{W}$ is around a certain value. Accordingly we set the relaxation parameter w as 0.00645, which is appropriate to guarantee the convergence.

Fig. 2 and Fig. 3 show the BER performance comparison between the conventional Neumann series approximation algorithm [9] and proposed signal detection algorithm with different initial solution, where i denotes the number of iterations. It is clear that the BER performance of both the conventional Neumann series approximation algorithm and the proposed signal detection algorithm improves with the number of iterations increasing. However the proposed algorithm outperforms the conventional one when the same

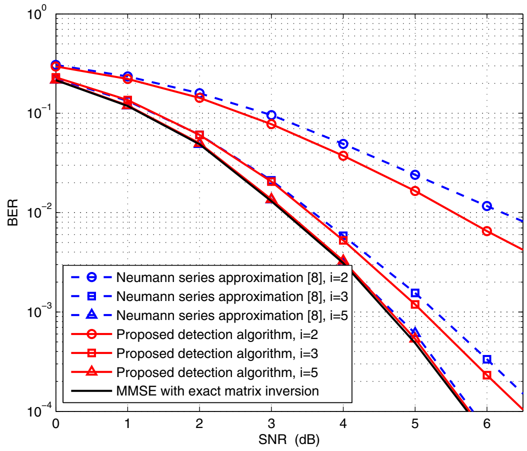

Fig. 2. BER performance comparison between the conventional Neumann series approximation algorithm and the proposed signal detection with zero-vector initial solution.

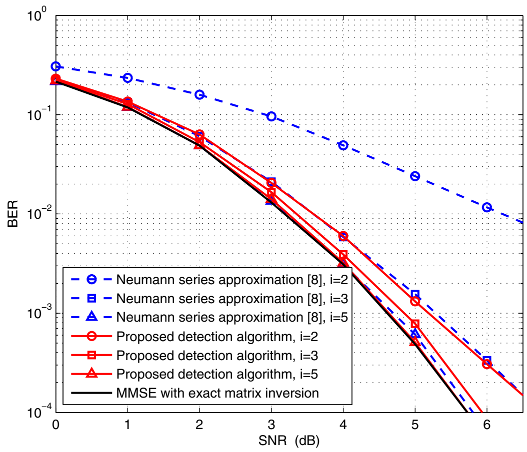

Fig. 3. BER performance comparison between the conventional Neumann series approximation algorithm and the proposed signal detection with zone-based initial solution.

number of iterations is used even if we choose zero vector as the initial solution, which indicates that a faster convergence rate can be achieved by the proposed algorithm.

Additionally, comparing the BER performance in Fig. 2 and Fig. 3, we can conclude that the proposed zone-based initial solution can accelerate the convergence rate remarkably. For example, the proposed zone-based initial solution outperforms the traditional one especially when the number of iterations i is small, which implies that the zone-based initial solution ensures a fast convergence rate of the Richardson method. What is more, when $i = 3$, the BER performance of proposed algorithm with zone-based initial solution is almost the same as that with traditional zero vector initial solution when $i = 5$, which implies the required times of iterations to achieve a certain estimation accuracy can be reduced, so the complexity can be reduced further. More importantly, when the number of iterations is large (e.g. $i = 5$ in Fig. 2 and Fig. 3), the proposed algorithm without the complicated matrix inversion can achieve the near-optimal BER performance of the MMSE algorithm with exact matrix, which verify the near-optimal performance of the proposed signal detection algorithm.

V. Conclusions

By fully exploiting the special property that the MMSE filtering matrix in large-scale MIMO systems is symmetric positive definite, we propose a low-complexity near-optimal signal detection algorithm based on the Richardson method to avoid the complicated matrix inversion, which can reduce the complexity from $\mathcal{O}(K^3)$ to $\mathcal{O}(K^2)$. We also prove the convergence of the proposed algorithm for any initial solution when the relaxation parameter is appropriate. To further accelerate the convergence rate and reduce the computational complexity, we propose a zone-based initial solution to the Richardson method. Simulation results verify that the proposed algorithm outperforms the conventional method, and achieves the near-optimal performance of the classical MMSE algorithm.

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