Optimal Deployment of Small Cell for Maximizing Average Sum Rate in Ultra-dense Networks

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Abstract—In future 5G communication system, demands of high data rate can be effectively met by ultra-dense networks (UDN), where a lot of small cells are deployed within the conventional macro cells. In this paper, we derive the optimal density of small cells in UDN to maximize the average sum rate (ASR). Specifically, based on the stochastic geometry, we use the homogeneous Poisson point process (PPP) to characterize the random distribution of both user equipments (UEs) and small cells. Then, the closed-form successful transmission probabilities in both uplink and downlink transmissions are derived by Laplace transformation. After that, we obtain ASR of small cells, and the maximization problem of ASR is analyzed under the constraints of outage probabilities. Based on convex optimization, the optimal density of small cell BS for maximizing the ASR of small cells is evaluated in a closed form. We also investigate the impact of system parameters on the optimal small cell BS density. Simulation results demonstrate that different maximized ASRs can be reached under different constraints from cellular networks, and the small cell performance is also influenced by the interference from macro cell UEs and small cells.

Index Terms—Ultra-dense networks (UDN), small cells, average sum rate, stochastic geometry, convex optimization.

I. INTRODUCTION

The rapid growth of mobile traffic in recent years drives the fast development of wireless communication, and future 5G system is expected to provide 1000 times higher network throughput and 1Gb/s mobile access rate [1]. Such an explosion of mobile traffic causes the shortage of spectrum resources. In order to solve this problem, ultra-dense networks (UDN) has been proposed to effectively alleviate the traffic demand of macro base stations (BSs) by densely deploying a large number of small cells in the networks [2]. UDN also introduces other benefits including efficient reuse of spectrum, improved data rate, and so on, which make it becomes one of the promising key technologies for future 5G system.

However, due to the aggravated interference generated by dense small cells, the performance of macro cell networks will be degraded, which results in a big challenge to the optimal deployment of small cells in UDN [3]. In order to reduce such interference, some solutions have been proposed to enhance the network performance [4]–[8], where the transmission rate is a key performance metric of UDN. Specifically, based on the interference statistics between macro and small cells, the transmission rate of UDN is evaluated for both dedicated and shared spectrum access under fixed grid-like macro cells [4]. The transmission rate is further optimized by using subchannel and power allocation [5], downlink hierarchical competition [6], and joint BS association with power control [7], etc., where all optimization works are self-organized and non-cooperative. When cooperative resource allocation is considered, the optimization of transmission rate for small cells in a single macro BS is investigated in [8]. However, it is insufficient to just consider one or several macro cells with fixed locations in UDN. For practical UDN, it is necessary to analyze the transmission rate of large-scale networks with the random deployment of macro cell users and small cells.

In this paper, the random distribution of macro cell users and small cell BSs is considered to derive the optimal deployment of small cells in UDN1. Specifically, based on the stochastic geometry, we use the homogeneous Poisson point process (PPP) to characterize the UDN with small cells sharing uplink frequency resources. Then, we derive the successful transmission probabilities of both uplink and downlink transmissions in small cells. With the help of convex optimization, we further analyze the average sum rate (ASR) maximization problem of small cells under the constraints of outage probabilities. Finally, the optimal small cell BS density for maximizing ASR of small cells are derived in closed form. Simulation results show that different maximized ASRs can be reached under different constraints from cellular networks. In addition, we show that the small cell performance is not only constrained by the transmission of macro cell, but also influenced by the interference from macro cell user equipments (UEs) and small cells.

The rest of this paper is organized as follows. Section II briefly describes the system model of UDN. Section III presents the successful transmission probabilities in UDN and the ASR of small cells. Section IV shows the optimization analysis of small cell BS density for the maximum ASR of small cells. Simulation results are provided in Section V. Finally, our conclusions are summarized in Section VI.

1Simulation codes are provided to reproduce the results presented in this paper: http://oa.ee.tsinghua.edu.cn/%7Edailinglong/.
II. SYSTEM MODEL

As shown in Fig. 1, we consider a UDN that small cells are densely deployed within the conventional macro cells. The uplink frequency resources of the macro cells are shared by small cells. Macro cell UEs are modeled as an independent homogeneous Poisson point process (PPP) $\Phi_M$ with density $\lambda_M$ on the two-dimensional plane $\mathbb{R}$. Similarly, the small cell BSs satisfy an independent homogeneous PPP $\Phi_S$ with density $\lambda_S$. The traffic in the UDN is assumed as full buffer. The powers of each macro cell UE, small cell BS and UE are defined as $P_M$, $P_{Sd}$, and $P_{Su}$, respectively.

By denoting the uplink transmission probability as $Pr_u$ ($0 \leq Pr_u \leq 1$), the downlink transmission probability $Pr_d$ should be $Pr_d = 1 - Pr_u$ ($0 \leq Pr_d \leq 1$). With the help of stochastic geometry, a typical receiver of small cells is assumed to be located at the origin on $\mathbb{R}$ [9]. If this typical receiver is receiving signals in downlink, it is a small cell UE, while this typical receiver is a small cell BS when it is receiving signals in uplink. Then, according to Palm theory [10], this typical receiver does not influence statistics of the PPP. Similarly, as the uplink frequency resources of macro cell networks are reused by small cells, the analysis will be performed on a typical receiver, which is a macro cell BS in the UDN.

The propagation channel in the UDN includes both path loss and Rayleigh fading with the form $P_t = P_t \delta_{t,r} R_{t,r}^{-\alpha}$, where $P_t$ and $P_r$ denote the power of transmitter and receiver, respectively. $R_{t,r}$ is the distance between the transmitter and receiver, and $\alpha$ denotes the path loss exponent with $\alpha \geq 2$. $\delta_{t,r}$ stands for Rayleigh fading coefficient, which follows an exponential distribution with unit mean in UDN [10].

III. AVERAGE SUM RATE OF SMALL CELLS

In this section, the successful transmission probabilities in UDN are analyzed. Then, based on the uplink and downlink transmissions in small cells, the average sum rate (ASR) of small cells is also obtained.

A. Successful Transmission Probabilities in UDN

First, we consider the condition that the typical macro cell BS suffers the interferences from both macro cell networks and small cells in the UDN. Compared with the distances from other interfering small cells to the typical macro cell BS, the distance between the small cell UE and its serving BS can be neglected because the coverage of each small cell is small in UDN [11]. Therefore, the signal-to-interference ratio (SIR) at the typical macro cell BS $SIR_M$ is

$$SIR_M = \frac{P_{M} \delta_{M0} R_{M0}^{-\alpha}}{\sum_{j \in \Phi_M} P_{M} \delta_{Mj} R_{Mj}^{-\alpha} + \sum_{k \in (\Phi_S \cap A)} P_{Sd} \delta_{Sk} R_{Sk}^{-\alpha} + \sum_{k \in (\Phi_S \cap B)} P_{Su} \delta_{Sk} R_{Sk}^{-\alpha}}$$

(1)

where $\delta_{M0}$ denotes the Rayleigh fading coefficient and, $R_{M0}$ denotes the distance from the desired macro cell UE to the typical macro cell BS. Similarly, $\delta_{k0}$ and $R_{k0}$ represent the Rayleigh fading coefficient and the distance from node $k$ to the origin in the macro cell, while $\delta_{k0}$ and $R_{k0}$ are corresponding parameters of the node $k$ in small cells. We define the compact point sets $A$ and $B$ as $A = \{m | \text{node } m \text{ is the receiver in uplink transmission in small cells} \}$ and $B = \{n | \text{node } n \text{ is the receiver in downlink transmission in small cells} \}$. Then, the following Lemma presents the successful transmission probability of the typical macro cell BS in UDN.

**Lemma 1.** The successful transmission probability of the typical macro cell BS in UDN satisfies:

$$\Pr(SIR_M \geq \xi_M) = \exp\left\{-\lambda_M \eta_M - \lambda_S \eta_M \left[Pr_u \left(\frac{P_{Sd}}{P_M}\right)^{\frac{\xi_d}{\alpha}} + Pr_d \left(\frac{P_{Su}}{P_M}\right)^{\frac{\xi_u}{\alpha}}\right]\right\}$$

(2)

where $Pr(\bullet)$ represents the probability, $\xi_M$ is the SIR threshold of uplink transmission of macro cell UE, $\eta_M = \pi R_{M0}^2 \xi_M^2$, $\Gamma(1 + \frac{\xi_d}{\alpha}) \Gamma(1 - \frac{\xi_u}{\alpha})$; $\Gamma(\bullet)$ denotes the gamma function with the form $\Gamma(z) = \int_0^\infty e^{-t^2} t^{z - 1} dt$.

**Proof:** Similar proof can be found in [10] with the help of Laplace transformation, so the detail is omitted here due to space limit.

Similarly, by denoting $SIR_{Su}$ as the Rayleigh fading coefficient and the distance from the typical small cell UE to its serving small cell BS, we have the following lemma.

**Lemma 2.** The successful transmission probability of the typical small cell UE in uplink transmission satisfies:

$$\Pr(SIR_{Su} \geq \xi_{Su}) = \exp\left\{-\lambda_M \eta_{Su} \left(\frac{P_{M}}{P_{Su}}\right)^{\frac{\xi_u}{\alpha}} - \lambda_S \eta_{Su} \left[Pr_u + Pr_d \left(\frac{P_{Su}}{P_M}\right)^{\frac{\xi_u}{\alpha}}\right]\right\}$$

(3)

where $\eta_{Su} = \pi R_{Su0}^2 \xi_{Su}^2 \Gamma(1 + \frac{\xi_u}{\alpha}) \Gamma(1 - \frac{\xi_u}{\alpha})$, and $\xi_{Su}$ is the SIR thresholds of uplink transmission in small cell.

In addition, by denoting $SIR_{Sd}$ and $R\xi_{Sd}$ as the Rayleigh fading coefficient and the distance from the typical small cell BS to its desire small cell UE, we have the following lemma.

**Lemma 3.** The successful transmission probability of the typical small cell BS in downlink transmission satisfies:

$$\Pr(SIR_{Sd} \geq \xi_{Sd}) = \exp\left\{-\lambda_M \eta_{Sd} \left(\frac{P_{M}}{P_{Sd}}\right)^{\frac{\xi_d}{\alpha}} - \lambda_S \eta_{Sd} \left[Pr_u \left(\frac{P_{Sd}}{P_M}\right)^{\frac{\xi_d}{\alpha}} + Pr_d\right]\right\}$$

(4)

where $\eta_{Sd} = \pi R_{Sd0}^2 \xi_{Sd}^2 \Gamma(1 + \frac{\xi_d}{\alpha}) \Gamma(1 - \frac{\xi_d}{\alpha})$, and $\xi_{Sd}$ is the SIR thresholds of downlink transmission in small cell.
B. Average Sum Rate in Small Cells

In UDN, the ASR in small cells is defined as [12]:

\[ f_{Su}(\lambda_S) = W_P u \lambda_S \log_2 (1 + \xi_{Su}) \times \exp \left\{ -\lambda_M \eta_{Su} \left( \frac{P_{Su}}{W_P} \right)^{\frac{2}{3}} - \lambda_S \eta_{Su} \left[ P_u + P_d \left( \frac{P_{Su}}{W_P} \right)^{\frac{2}{3}} \right] \right\}, \]

where \( W \) is the bandwidth of macro cell uplink transmission, and this bandwidth is also reused by small cells. Then, we have the following definition:

**Definition 1.** The ASR of uplink transmission in small cells is

\[ f_{Su}(\lambda_S) = W_P u \lambda_S \log_2 (1 + \xi_{Su}) \times \exp \left\{ -\lambda_M \eta_{Su} \left( \frac{P_{Su}}{W_P} \right)^{\frac{2}{3}} - \lambda_S \eta_{Su} \left[ P_u + P_d \left( \frac{P_{Su}}{W_P} \right)^{\frac{2}{3}} \right] \right\}, \]

while the ASR of downlink transmission in small cells is

\[ f_{Sd}(\lambda_S) = W_P d \lambda_S \log_2 (1 + \xi_{Sd}) \times \exp \left\{ -\lambda_M \eta_{Sd} \left( \frac{P_{Sd}}{W_P} \right)^{\frac{2}{3}} - \lambda_S \eta_{Sd} \left[ P_u + P_d \left( \frac{P_{Sd}}{W_P} \right)^{\frac{2}{3}} \right] \right\}. \]

Thus, the ASR of small cells in UDN \( f(\lambda_S) \) is

\[ f(\lambda_S) = f_{Su}(\lambda_S) + f_{Sd}(\lambda_S). \]

IV. OPTIMIZATION OF ASR FOR SMALL CELLS

When small cells reuse the frequency resources of macro cell networks, the reliable transmission of macro cells cannot be disturbed, i.e., the transmission of small cells must guarantee the outage probabilities at the macro cell BS in UDN. Considering the transmission in small cells is divided as uplink and downlink transmissions, we have the following four constraints:

\[ 0 \leq \lambda_S \leq \lambda_{S,\text{max}}, \]

\[ 1 - e^{-\lambda_M \eta_{Sd} - \lambda_S \eta_{Sd} \left[ P_u + P_d \left( \frac{P_{Sd}}{W_P} \right)^{\frac{2}{3}} \right]} \leq \theta_{Sd}, \]

\[ 1 - e^{-\lambda_M \eta_{Su} - \lambda_S \eta_{Su} \left[ P_u + P_d \left( \frac{P_{Su}}{W_P} \right)^{\frac{2}{3}} \right]} \leq \theta_{Su}, \]

\[ 1 - e^{-\lambda_M \eta_{Sd} - \lambda_S \eta_{Sd} \left[ P_u + P_d \left( \frac{P_{Sd}}{W_P} \right)^{\frac{2}{3}} \right]} \leq \theta_{Sd}. \]

From inequality (12), we have

\[ \lambda_S \leq \frac{-\lambda_M \eta_{Su} - \lambda_S \eta_{Su} \left[ P_u + P_d \left( \frac{P_{Su}}{W_P} \right)^{\frac{2}{3}} \right]}{\theta_{M}} = \lambda_{S,\sup 1}. \]

From inequality (13), we have

\[ \lambda_S \leq \frac{-\lambda_M \eta_{Sd} - \lambda_S \eta_{Sd} \left[ P_u + P_d \left( \frac{P_{Sd}}{W_P} \right)^{\frac{2}{3}} \right]}{\theta_{Sd}} = \lambda_{S,\sup 2}. \]

From inequality (14), we have

\[ \lambda_S \leq \frac{-\lambda_M \eta_{Sd} - \lambda_S \eta_{Sd} \left[ P_u + P_d \left( \frac{P_{Sd}}{W_P} \right)^{\frac{2}{3}} \right]}{\theta_{Sd}} = \lambda_{S,\sup 3}. \]
where \(\lambda_S^*\) satisfies
\[
\lambda_S^* = \begin{cases} 
\lambda_{S,l}, & f'(\lambda_S) < 0 \text{ when } \lambda_S \in (\lambda_{S,l}, \lambda_{S,h}), \\
\lambda_{S}^*, & f'(\lambda_{S}^*) = 0 \text{ and } \lambda_{S}^* \in (\lambda_{S,l}, \lambda_{S,h}), \\
\lambda_{S,h}, & f'(\lambda_S) > 0 \text{ when } \lambda_S \in (\lambda_{S,l}, \lambda_{S,h}), 
\end{cases}
\]
and \(\lambda_{S,l}\) and \(\lambda_{S,h}\) are the lower and upper limits of the feasible region of small cell BS density. We can see the feasible region boundary from these two figures, we can find that because the distance of uplink transmission is shorter than that of downlink transmission, which makes the uplink signals suffer less propagation loss than the downlink signals. In addition, \(\lambda_{S}^*\) is the maximum value when \(\lambda_{S}^* = \arg\max_{\lambda_S \in \mathbb{R}} f(\lambda_S)\).

**Proof:** The first derivative of \(f(\lambda_S)\) is
\[
f'(\lambda_S) = A_1 e^{-A_2 - \lambda_S A_3} (1 - \lambda_S A_3) + B_1 e^{-B_2 - \lambda_S B_3} (1 - \lambda_S B_3).
\]

First, if \(f'(\lambda_S) < 0\), then according to Proposition 1, we can know that \(f(\lambda_S)\) can get the maximum value as \(\lambda_S = \lambda_{S,l}\) since \(f(\lambda_S)\) monotonically decreases in \((\lambda_{S,l}, \lambda_{S,h})\). Second, if \(f'(\lambda_S) > 0\), \(f(\lambda_{S,h})\) is the maximum value as \(f(\lambda_S)\) is a continuously bounded function in the close set \([\lambda_{S,l}, \lambda_{S,h}]\), and if \(\exists \lambda_{S}^* \in (\lambda_{S,l}, \lambda_{S,h})\) which leads to \(f'(\lambda_{S}^*) = 0\), \(f(\lambda_{S}^*)\) must be the local maximum or minimum value in \([\lambda_{S,l}, \lambda_{S,h}]\). So \(\lambda_{S}^*\) is the maximum value when \(\lambda_{S}^* = \arg\max_{\lambda_S \in \mathbb{R}} f(\lambda_S)\).

Last, combine those three points above, we get \(\lambda_S^*\), then consider the constraints of power and the outage probabilities of both macro and small cell transmissions, the optimal small cell density \(\lambda_{S}^\text{max}\) for maximizing ASR of small cells is obtained as equation (18).

### V. Simulation Results

In this section, we evaluate ASR of small cells with the main simulation parameters provided in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Physical Meaning</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>Path loss</td>
<td></td>
</tr>
<tr>
<td>(\lambda_M)</td>
<td>Macro cell UE density</td>
<td>(1 \times 10^{-6}) macro cell UE/m²</td>
</tr>
<tr>
<td>(\lambda_S)</td>
<td>Small cell BS density</td>
<td>(1 \times 10^{-6}) small cell BS/m²</td>
</tr>
<tr>
<td>(P_M)</td>
<td>The power of macro cell UE</td>
<td>35 dBm</td>
</tr>
<tr>
<td>(P_S)</td>
<td>The power of small cell UE</td>
<td>25 dBm</td>
</tr>
<tr>
<td>(P_{S,d})</td>
<td>The power of small cell BS</td>
<td>30 dBm</td>
</tr>
<tr>
<td>(P_{S,u}/P_{S,d})</td>
<td>The probability of uplink/downlink communication in a small cell</td>
<td>0.6/0.4</td>
</tr>
<tr>
<td>(\xi_M)</td>
<td>SIR threshold of the macro cell uplink transmission</td>
<td>-5 dB</td>
</tr>
<tr>
<td>(\xi_{S,u}/\xi_{S,d})</td>
<td>SIR threshold of the small cell uplink/downlink transmission</td>
<td>0 dB</td>
</tr>
<tr>
<td>(R_{M})</td>
<td>The distance from macro cell BS to the typical cellular user</td>
<td>100m</td>
</tr>
<tr>
<td>(R_{S,u}/R_{S,d})</td>
<td>The uplink/downlink distance</td>
<td>50m/60m</td>
</tr>
</tbody>
</table>

Fig. 2 (a) and (b) show the ASR of small cells with two different feasible regions of small cell BS density. We can see the maximum ASR of small cells locates in a interval of small cell BS density, where the two boundary values of that interval correspond to the maximum uplink ASR and downlink ASR. This phenomenon verifies Proposition 1 before. When the small cell BS density is not very high, the ASR of small cells is increasing as the small cell BS density becomes high, which means more small cell BSs will bring larger performance gain. However, as the small cell BS density continues to increase, the interference caused by small cells in the UDN cannot be neglected, so the ASR becomes to fall down. In Fig. 2(a), the powers of small cell UE and BS are set as 14 dBm and 11 dBm, we can see the downlink ASR of small cells is lower than the uplink ASR, since the receivers in downlink transmission of small cells cannot endure much interference than uplink receivers. In Fig. 2 (b), the powers of small cell UE and BS are set as 25 dBm and 30 dBm, respectively, and we can see the downlink ASR is larger than the uplink ASR, but the growth speed of downlink ASR is slower than that of the uplink ASR when small cell BS density is low. This is caused by the fact that the distance of uplink transmission is shorter than that of downlink transmission, which makes the uplink signals suffer less propagation loss than the downlink signals. In addition, by comparing these two figures, we can find that because the power of small cell UE and BS in Fig. 2 (a) is low, macro cells can allow more small cells to reuse their frequency resources, then we can see the feasible region of small cell BS density is wide and the maximum ASR of small cells can be obtained.

Fig. 3 shows the maximum ASR of small cells against the macro cell UE density and power of macro cell UE. From Fig. 3 (a), we can see the maximum ASR of small cells is decreasing as the macro cell UE density becomes high. This is because high macro cell UE density will cause more interference to the small cells, and the constraints to small cells become more strict. When the macro cell UE density is high enough, the receivers in small cells are severely affected by the interference from macro cell UEs, which leads ASR come to zero. It can also be known that high-power macro cell UE will cause more interference to small cells, so the maximum ASR of small cells is lower while the power of macro cell UE is higher. In Fig. 3 (b), it can be seen that the maximum ASR of small cell is increasing as the power of macro cell UE
increases. This can be explained by the fact that high power of macro cell UE can endure more interference from small cells, more small cell BSs can be deployed into the networks. As the power continues to increase, the interference generated by macro cells becomes larger, then the maximum ASR of small cells begins to decrease. In addition, lower macro cell UE density can lead maximum ASR of small cells to be larger. This is because the interference from macro cell UEs to the small cells becomes smaller.

Fig. 3. The maximum ASR of small cells vs. Density and power of macro cell UE.

Finally, Fig. 4 illustrates the relationship among the maximum ASR, uplink power, and downlink power of small cells. We can see the performance gain in the uplink transmission is larger than that in the downlink transmission of small cells. This can be explained as follows: 1) The uplink transmission distance is shorter than the downlink transmission distance, which makes the signal experienced less propagation loss in the uplink; 2) The probabilities of uplink and downlink transmission are set as 0.6 and 0.4, respectively, which means the number of links in uplink transmission is more than that in the downlink transmission, so we can see that the uplink transmission dominates the whole performance gain. An extreme example in Fig. 4 is when uplink and downlink transmission are 5 dBm and 35 dBm, the maximum ASR of small cells is very low, because not only the uplink transmission cannot bring much performance gain to the small cells, but also the downlink transmission will generate serious interference to the whole networks.

VI. CONCLUSIONS

In this paper we have optimized the small cell BS density in UDN for maximizing ASR of small cells. By modeling the whole networks as the homogeneous PPPs, we have investigated the successful transmission probabilities of both macro cell networks and small cells. The ASR expression of small cells was evaluated, and an optimization problem of small cell BS density was formulated by considering outage probabilities and density constraints. Then, we proved that the maximum ASR of small cells locates in a fixed interval of small cell BS density. The boundaries of the density were also shown. Finally, the optimal small cell density was derived in closed form for maximizing ASR of the small cells in UDN. The impact of the parameters such as optimal small cell BS density and macro cell UE power were discussed through simulation results, which verify that the maximum ASR of small cells is influenced by both the constraints from macro cell networks and the interference in UDN.

REFERENCES