Spatially correlated channel estimation based on block iterative support detection for massive MIMO systems

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Downlink channel estimation with low pilot overhead is an important and challenging problem in massive multiple-input–multiple-output (MIMO) systems due to the substantially increased MIMO channel dimension. A block iterative support detection (block-ISD)-based algorithm for downlink channel estimation to reduce the pilot overhead is proposed, which is achieved by fully exploiting the block sparsity inherent in the block-sparse equivalent channel derived from the spatial correlations of MIMO channels. Furthermore, unlike conventional compressive sensing (CS) algorithms that rely on prior knowledge of the sparsity level, block-ISD relaxes this demanding requirement and is thus more practically appealing. Simulation results demonstrate that block-ISD yields better normalised mean square error (NMSE) performance than classical CS algorithms, and achieve a reduction of 84% pilot overhead compared with conventional channel estimation techniques.

Introduction: Recently, massive multiple-input–multiple-output (MIMO) systems with a large number of antennas at the base station (BS) have emerged as a key promising technology for future fifth-generation wireless communications. It has been proved that massive MIMO systems can reduce transmit power as well as increase spectrum efficiency by orders of magnitude [1]. In such systems, accurate downlink channel state information is essential for channel adaptive techniques such as water filling, beamforming and so on. As the number of antennas keeps increasing in massive MIMO systems, efficient low-overhead channel estimation will be an increasingly important and challenging problem. Conventional channel estimation techniques including least square (LS) and minimum mean square error (MMSE) [2] are not suitable for massive MIMO systems due to the number of required orthogonal pilot scales linearly with the number of antennas at the BS, which results in prohibitively high pilot overhead. Recently, several efficient channel estimation schemes based on compressive sensing (CS) have been proposed to reduce pilot overhead by taking channel sparsity (i.e. the channel is sparse) into account [3–5]. However, these CS-based channel estimation schemes usually assume prior knowledge of the sparsity level, i.e. the number of non-zero elements of the channel impulse response (CIR), which is usually unknown and difficult to be accurately estimated in practice. In addition, due to the physical propagation characteristics of multiple antennas, CIRs associated with different antennas inevitably share spatial correlations [6], which have not been considered by the existing CS-based channel estimation schemes to further improve the performance.

Building on the iterative support detection (ISD) algorithm [7], in this Letter we propose an improved block ISD (block-ISD)-based algorithm for downlink channel estimation to reduce the pilot overhead. Specifically, by taking into account the spatial correlations of MIMO channels caused by the physical propagation characteristics of multiple antennas and close antenna spacing at the BS, we generate the block-sparse equivalent CIR with the preferred block sparsity. Accordingly, we propose a block-ISD-based algorithm to exploit the block sparsity to significantly reduce the pilot overhead required for accurate channel estimation. Note that block-ISD requires no prior knowledge of the sparsity level, and is thus more practically appealing than conventional CS-based algorithms.

System model: We consider a massive MIMO system with N_{T} antennas at the BS and K scheduled single-antenna users ($N_{\mathrm{T}} \gg K$) using the commonly used orthogonal frequency division multiplexing (OFDM) modulation. Normally, frequency-domain pilots are used for channel estimation in OFDM-based systems [3–5]. At the user side, the received pilots in the frequency domain can be expressed as

$$\boldsymbol{y}_{\Omega} = \sum_{i=1}^{N_{\mathrm{T}}} \boldsymbol{C}_i (\boldsymbol{F}_L)_{\Omega} \boldsymbol{h}_i + \boldsymbol{n}_{\Omega} \tag{1}$$

where Ω is the index set of subcarriers assigned to pilots, which can be randomly selected from the subcarrier set $[1, 2, \ldots, N]$. $\boldsymbol{C}_i = \mathrm{diag}\, \boldsymbol{c}_i$ with $\boldsymbol{c}_i \in \mathcal{C}^{p\times 1}$ being the pilot vector for the ith transmit antenna, and the number of pilots is p. $\boldsymbol{F}_L \in \mathcal{C}^{N\times L}$ is a sub-matrix consisting of the first L columns of the discrete Fourier transform matrix of size $N \times N$, where N is the OFDM symbol length. $(\boldsymbol{F}_L)_{\Omega}$ is the sub-matrix consisting of rows of $\boldsymbol{F}_L$ with indexes from Ω. $\boldsymbol{h}_i = [\boldsymbol{h}_i(1), \boldsymbol{h}_i(2), \ldots, \boldsymbol{h}_i(L)]^{\mathrm{T}}$ denotes the CIR between the ith transmit antenna of the BS and the single receive antenna of the user with the maximum channel length L. $\boldsymbol{n}_{\Omega} = [n_1, \ldots, n_p]^{\mathrm{T}}$ represents the noise vector consisting of independent and identically distributed additive white complex Gaussian noise variables with zero mean and unit variance. For simplicity, (1) can also be written as

$$\boldsymbol{y}_{\Omega} = \boldsymbol{P}\boldsymbol{h} + \boldsymbol{n}_{\Omega} \tag{2}$$

where $\boldsymbol{P} = [\boldsymbol{C}_1(\boldsymbol{F}_L)_{\Omega}, \boldsymbol{C}_2(\boldsymbol{F}_L)_{\Omega}, \ldots, \boldsymbol{C}_{N_{\mathrm{T}}}(\boldsymbol{F}_L)_{\Omega}]$ can be regarded as the sensing matrix, $\boldsymbol{h} = [\boldsymbol{h}_1^{\mathrm{T}}, \boldsymbol{h}_2^{\mathrm{T}}, \ldots, \boldsymbol{h}_{N_{\mathrm{T}}}^{\mathrm{T}}]^{\mathrm{T}}$ is the aggregate CIR from N_{T} antennas to be estimated in massive MIMO systems.

Downlink channel estimation based on block-ISD: In this Section, we first generate the block-sparse equivalent CIR and then propose an improved block-ISD algorithm for channel estimation.

Generation of block-sparse equivalent CIR: Owing to the physical propagation characteristics of multiple antennas and close antenna spacing at the BS, CIRs $\{\boldsymbol{h}_i\}_{i=1}^{N_{\mathrm{T}}}$ associated with different transmit antennas have similar path arrival times, and thus they share a common support [6], i.e. $\boldsymbol{\Gamma}_{\boldsymbol{h}_1} = \boldsymbol{\Gamma}_{\boldsymbol{h}_2} = \cdots = \boldsymbol{\Gamma}_{\boldsymbol{h}_{N_{\mathrm{T}}}}$, where $\boldsymbol{\Gamma}_{\boldsymbol{h}_i} = \{k : \boldsymbol{h}_i(k) \neq 0\}$ denotes the support of $\boldsymbol{h}_i$. Since the CIRs from different transmit antennas share a common support, we can group the elements of $\boldsymbol{h}_i$ with the same indexes into non-zero blocks and zero blocks to generate the block-sparse equivalent CIR $\boldsymbol{g} = [\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_L]^{\mathrm{T}}$. More specifically, the relationship between $\boldsymbol{h}$ and $\boldsymbol{g}$ can be denoted by

$$\boldsymbol{g}((l-1)N_{\mathrm{T}} + n_t) = \boldsymbol{h}((n_t - 1)L + l) \tag{3}$$

where $l = 1, 2, \ldots, L$ and $n_t = 1, 2, \ldots, N_{\mathrm{T}}$. It is important that if we equally divide $\boldsymbol{g}$ into L blocks with N_{T} elements in each block, these N_{T} continuous elements in the lth block $\boldsymbol{g}_l$ are all zeros or non-zeros. Thus, the generated block-sparse equivalent CIR $\boldsymbol{g}$ enjoys the preferred property of 'block sparsity'. This implies that we can treat the N_{T} continuous elements of the support $\boldsymbol{\Gamma}_g$ of $\boldsymbol{g}$ as a whole and update them simultaneously.

Accordingly, similar to (3), we can obtain a new sensing matrix $\boldsymbol{\Theta}$ by rearranging the columns of $\boldsymbol{P}$ in (2) as

$$\boldsymbol{\Theta}(:, (l-1)N_{\mathrm{T}} + n_t) = \boldsymbol{P}(:, (n_t - 1)L + l) \tag{4}$$

Therefore, the channel estimation problem (2) can be reformulated as

$$\boldsymbol{y}_{\Omega} = \boldsymbol{\Theta}\boldsymbol{g} + \boldsymbol{n}_{\Omega} \tag{5}$$

This is an underdetermined problem with $\boldsymbol{g}$ of size $N_{\mathrm{T}}L \times 1$ and $\boldsymbol{y}_{\Omega}$ of size $p \times 1$, where p is usually much smaller than $N_{\mathrm{T}}L$ due to the large number of antennas and the limited pilot overhead. Traditional channel estimation techniques such as LS and MMSE cannot recover $\boldsymbol{g}$ with limited pilot overhead. In this Letter, the block sparsity inherent in the generated block-sparse equivalent CIR $\boldsymbol{g}$ will be utilised by the proposed block-ISD algorithm in the following Section to solve this problem.

Downlink channel estimation based on block-ISD

Algorithm 1 Block-ISD algorithm

Input:
  1) Measurements $\boldsymbol{y}_{\Omega}$; 2) Sensing matrix Θ
1: Initialisation: $S = 0$ and $\boldsymbol{\Gamma}_g^{(0)} = \emptyset$
2: **while** $\mathrm{Card}(\boldsymbol{\Gamma}_g^{(s)}) < N_T L - p$ **do**
3: $\quad W^{(s)} = (\boldsymbol{\Gamma}_g^{(s)})^{c}$;
4: $\quad \boldsymbol{g}^{(s)} \leftarrow \min_{\boldsymbol{g}^{(s)}} \|\boldsymbol{g}_{W^{(s)}}^{(s)}\|_1$ s.t. $\boldsymbol{y}_{\Omega} = \boldsymbol{\Theta}\boldsymbol{g}^{(s)} + \boldsymbol{n}_{\Omega}$;
5: $\quad \boldsymbol{v}^{(s)} = \mathrm{Sort}(\boldsymbol{g}^{(s)})$;
6: $\quad i \leftarrow \min_i$ s.t. $|\boldsymbol{v}^{(s)}(i+1)| - |\boldsymbol{v}^{(s)}(i)| > |\tau^{(s)}|$;
7: $\quad \epsilon^{(s)} = |\boldsymbol{v}^{(s)}(i)|$;
8: $\quad \boldsymbol{\Gamma}_v^{(s)} = \{k \text{ s.t. } |\boldsymbol{v}^{(s)}(k)| > \epsilon^{(s)}\}$;
9: $\quad \boldsymbol{\Gamma}_g^{(s)} = \{(l-1)N_T + 1{:}1{:}lN_T \text{ s.t. } \mathrm{Card}(\{(l-1)N_T + 1:1: lN_T\} \cap \boldsymbol{\Gamma}_v^{(s)}) > N_T/2\}$;
10: $\quad s = s + 1$.
11: **end while**
12: **return** $\hat{\boldsymbol{g}} = \boldsymbol{g}^{(s)}$
Output:
  Recovered block-sparse equivalent CIR $\hat{\boldsymbol{g}}$

The pseudocode of block-ISD is given in Algorithm 1. Note that block-ISD updates all the elements of the recovered signal $\boldsymbol{g}^{(s)}$ in the sth iteration through solving the truncated basic pursuit (BP) problem [8] in step 4

$$\min_{\boldsymbol{g}^{(s)}} \|\boldsymbol{g}^{(s)}_{W^{(s)}}\|_1 \ \text{s.t.}\ \boldsymbol{y}_\Omega = \boldsymbol{\Theta}\boldsymbol{g}^{(s)} + \boldsymbol{n}_\Omega \tag{6}$$

where $\|\boldsymbol{g}^{(s)}_{W^{(s)}}\|_1 = \sum_{w \in W^{(s)}} |\boldsymbol{g}^{(s)}(w)|$. This problem can be efficiently solved by calling a BP algorithm such as YALL1 [7]. Then, the support $\boldsymbol{\Gamma}^{(s)}_g$ is updated in the sth iteration through the adjacent support detection in steps 5–9. In these five steps, we first sort $\boldsymbol{g}^{(s)}$ in an ascending order in step 5 to obtain $\boldsymbol{v}^{(s)}$. Then, the support of $\boldsymbol{v}^{(s)}$ can be detected based on the ‘first significant jump’ rule [7] in step 6, which searches for the smallest i that satisfies $|\boldsymbol{v}^{(s)}(i+1)| - |\boldsymbol{v}^{(s)}(i)| > |\tau^s|$, where $\tau^{(s)} = (LN_\mathrm{T})^{-1}\|\boldsymbol{v}^{(s)}\|_\infty$ [7]. The smallest i is the index where the ‘first significant jump’ occurs in an ascending ordered vector $\boldsymbol{v}^{(s)}$. Next, we set the threshold $\varepsilon^{(s)} = |\boldsymbol{v}^{(s)}(i)|$ in step 7, then the support of $\boldsymbol{v}^{(s)}$ can be updated in step 8 based on this threshold. Finally, due to the block sparsity of $\boldsymbol{g}^{(s)}$, the support of $\boldsymbol{g}^{(s)}$ can be updated in step 9, where Card$(\cdot)$ denotes the number of elements of a set.

Note that the support $\boldsymbol{\Gamma}^{(s)}_g$ is independent of $\boldsymbol{\Gamma}^{(s-1)}_g$ in block-ISD, which is different from the classical greedy CS algorithm called orthogonal matching pursuit (OMP) [9]. In OMP, only one element of $\boldsymbol{\Gamma}^{(s)}_g$ is updated in each iteration, and once an element is added to $\boldsymbol{\Gamma}^{(s)}_g$, this element will not be removed in the following iterations. From this aspect, block-ISD is similar to subspace pursuit (SP) [10] and compressive sampling matching pursuit (CoSaMP) [9]. They update all elements of the recovered signal in every iteration, whereby the support detection not only selects the desired elements but also removes the undesired elements. However, the support detection of SP and CoSaMP is based on the sparsity level assumed to be known *a priori*, whereas the support detection of block-ISD is based on the sparsity-independent threshold $\varepsilon^{(s)}$. Thus, block-ISD can recover the signal without prior knowledge of the channel sparsity level.

The key difference between ISD and block-ISD is the consideration of the block sparsity of $\boldsymbol{g}^{(s)}$. For a certain non-zero block of $\boldsymbol{g}$, the continuous N_T elements of this block are supposed to be all non-zeros, so their indexes are supposed to be included in $\boldsymbol{\Gamma}^{(s)}_g$. However, some indexes may be incorrectly detected due to the impact of noise. Nevertheless, we can determine whether this block is a zero block or a non-zero block by comparing the number of indexes included in $\boldsymbol{\Gamma}^{(s)}_g$ with $N_\mathrm{T}/2$ (half of the block length N_T) in step 9. Only when more than half of the indexes of a certain block are included in $\boldsymbol{\Gamma}^{(s)}_g$, will all N_T indexes of this block be added in $\boldsymbol{\Gamma}^{(s)}_g$. This mechanism considering the block sparsity is expected to increase the robustness of the support detection and thus improves the channel estimation performance as will be verified by the simulation results in the following Section.

Simulation results: We consider an $N_\mathrm{T} = 32$ massive MIMO system with the system bandwidth of 50 MHz and the OFDM symbol length $N = 4096$. We adopt the international telecommunication union Vehicular-A channel model [3] with the maximum channel length $L = 128$. Fig. 1 shows the normalised mean square error (NMSE) performance comparison between the proposed block-ISD and the classical ISD [7] and BP [8] algorithms, where the number of pilots is $p = 640$. In addition, the performance of the exact LS algorithm assuming the exact knowledge of the support of block-sparse equivalent CIR is also presented as the lower bound of NMSE for comparison. It can be observed that block-ISD outperforms both classical ISD and BP algorithms. Specifically, block-ISD achieves more than 4 dB signal-to-noise ratio (SNR) gain compared with ISD and BP algorithms when the target NMSE of 10^{-1} is considered. The performance gain is mainly attributed to the exploration of block sparsity inherent in the generated block-sparse equivalent CIR. Note that block-ISD obviously outperforms ISD when SNR is not very high. This is due to the fact that block-ISD is more capable of correcting the support detection error caused by the additive noise than ISD when SNR is not very high (e.g. SNR < 20 dB), which therefore enhances the support detection and ultimately leads to a lower NMSE. For conventional channel estimation techniques such as LS and MMSE [2], the number of pilots p should be as large as $LN_\mathrm{T} = 128 \times 32 = 4096$ to ensure (5) as an over determined problem. That is to say, block-ISD achieves a substantial reduction of $(4096 - 640)/4096 = 84\%$ pilot overhead compared with these conventional channel estimation techniques without considering the channel sparsity.

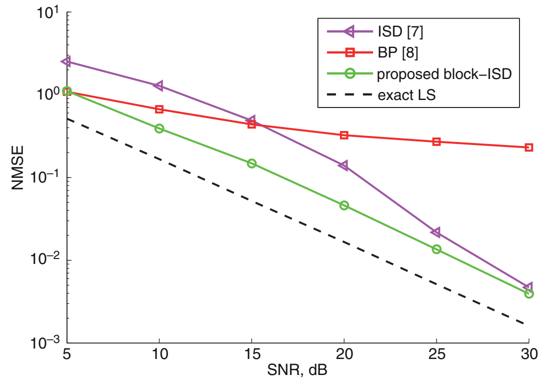

Fig. 1 *NMSE performance comparison between block-ISD, ISD and BP*

Conclusion: In this Letter, we have investigated the challenging problem of downlink channel estimation with acceptable pilot overhead for massive MIMO systems. It is found that by exploring the block sparsity inherent in the block-sparse equivalent CIR, which is generated by considering the spatial correlations of MIMO channels, the proposed block-ISD algorithm can improve the channel estimation performance by more than 4 dB compared with classical ISD and BP algorithms. In addition, we have shown that block-ISD requires no prior knowledge of the channel sparsity level, thereby making an important step towards practical implementation. Simulation results have demonstrated that block-ISD can achieve a reduction of 84% pilot overhead compared with the conventional channel estimation techniques. The extension of block sparsity to temporally correlated channels will be left as future work.

07 October 2014
doi: 10.1049/el.2014.3576
One or more of the Figures in this Letter are available in colour online.

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