align along the vector $[0\ 1\ 0\ 1\ 0\ 1\ 0\ 0\ 0]^T$, whereas $s_1^{[3]}$ and $s_1^{[4]}$ align along the vector $[0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0]^T$. This is due to the fact that each pair of these symbols has a nonzero entry in the beamforming vectors only when the receiver selects mode 2, which is guaranteed by step 7 in the BIA algorithm. Thus, the interfering signal occupies 6-D space. To ensure that all desired symbols are decodable, we need to prove that all vectors carrying the desired symbols are linearly independent, and the 3-D subspace carrying the desired symbols does not intersect with the 6-D interference subspace. This can be easily proved by showing that the 9×9 matrix $\mathbf{R}$ defined in (23), shown at the bottom of the previous page, which contains all the received desired and interference vectors, is full rank, which follows from the fact that all rows and columns in $\mathbf{R}$ are almost surely linearly independent. Thus, receiver 1 achieves 1/3 DoF almost surely. The same analysis can be applied to receivers 2–4. At every receiver, the interference will occupy 6-D space, whereas the desired signal occupies 3-D space. Hence, the achieved sum DoF is 4/3. Note that for $K = 3$, each two symbols can be aligned at one unintended receiver, and the proposed algorithm reduces to the scheme in [6], achieving a sum DoF of 6/5. For $K = 5$, each two symbols can be aligned at three unintended receivers, and the achieved sum DoF is 10/7.

V. CONCLUSION

In this paper, we have shown that linear BIA using staggered antenna mode switching can achieve a sum DoF of $2K/(K+2)$ in the K-user SISO interference channel. A key insight is that each signal dimension from one user can be aligned with a single set of distinct users at the receivers of the remaining users. This result suggests that we can double the unity DoF of orthogonal multiple-access schemes without channel state information at the transmitters. Moreover, we proposed an algorithm to generate the transmit beamforming vectors and antenna switching patterns utilized in BIA. We showed that the proposed algorithm can achieve the $2K/(K+2)$ sum DoF for any K. By applying this algorithm to the four-user interference channel, it was shown that a sum DoF of 4/3 is achievable.

REFERENCES

[1] S. A. Jafar and S. Shamai, "Degrees of freedom region for the MIMO X channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 151–170, Jan. 2008.
[2] S. A. Jafar, "Interference alignment: A new look at signal dimensions in a communication network," *Found. Trends Commun. Inf. Theory*, vol. 7, no. 1, pp. 1–134, Jun. 2011.
[3] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.
[4] S. A. Jafar, "Exploiting channel correlations—Simple interference alignment schemes with no CSIT," in *Proc. IEEE GLOBECOM*, Miami, FL, USA, Dec. 2010, pp. 1–5.
[5] T. Gou, C. Wang, and S. A. Jafar, "Aiming perfectly in the dark-blind interference alignment through staggered antenna switching," *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2734–2744, Jun. 2011.
[6] C. Wang, "Degrees of freedom characterization: The 3-user SISO interference channel with blind interference alignment," *IEEE Commun. Lett.*, vol. 18, no. 5, pp. 757–760, May 2014.
[7] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, "On the degrees of freedom of SISO interference and X channels with delayed CSIT," in *Proc. 49th Annu. Allerton Conf. Commun., Control, Comput.*, Monticello, IL, USA, Sep. 2011, pp. 625–632.
[8] M. Kang and W. Choi, "Ergodic interference alignment with delayed feedback," *IEEE Signal Process. Lett.*, vol. 20, no. 5, pp. 511–514, May 2013.
[9] Y. Tian and A. Yener, "Guiding the blind transmitters: Degrees of freedom optimal interference alignment using relays," *IEEE Trans. Inf. Theory*, vol. 59, no. 8, pp. 4819–4832, Aug. 2013.
[10] S. Jafar, "Blind interference alignment," *IEEE J. Sel. Topics Signal Process.*, vol. 6, no. 3, pp. 216–227, Jun. 2012.
[11] A. M. Alaa and M. H. Ismail, Online Appendix. [Online]. Available: http://arxiv.org/pdf/1408.6427v2.pdf

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Weighted-Graph-Coloring-Based Pilot Decontamination for Multicell Massive MIMO Systems

Xudong Zhu, Linglong Dai, Zhaocheng Wang, and Xiaodong Wang

***Abstract*—A multicell massive multiple-input multiple-output (MIMO) system, which utilizes a large number of base-station antennas to simultaneously serve a set of users, suffers from pilot contamination (PC) due to unavoidable reuse of pilots in adjacent cells. In this paper, a weighted-graph-coloring-based pilot decontamination (WGC-PD) scheme is proposed to mitigate PC for multicell massive MIMO systems. Specifically, based on limited cooperation among cells, an edge-weighted interference graph (EWIG) is first constructed to depict the potential PC relationship among users, whereby two users in different cells are connected by a weighted edge, indicating the strength of potential PC when they reuse the same pilot. Then, inspired by classical graph coloring algorithms, we develop the WGC-PD scheme by denoting each color as a pilot and each vertex as a user in the EWIG, which is able to mitigate PC by assigning different pilots to connected users with a large weight in a greedy way with insufficient pilot resource. Compared with exhaustive search among numerous pilot assignment solutions, the proposed WGC-PD scheme is able to mitigate PC with significantly reduced complexity, which is verified by numerical results.**

***Index Terms*—Edge-weighted interference graph (EWIG), graph coloring problem, massive multiple-input multiple-output (MIMO), pilot contamination (PC).**

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) has been investigated to meet the exponential increase in mobile traffic in future fifth-generation wireless systems [1]–[4], whereby a base station (BS) equipped with a large number of antennas serves multiple users simultaneously. When the number of BS antennas goes to infinity while the number of users is kept fixed, intracell interference and uncorrelated noise can be significantly reduced [1], [2]. In addition, massive MIMO systems are also beneficial in the context of detection and estimation problems via sensor networks [5], [6]. However, pilot contamination (PC) caused by the reuse of pilots in adjacent cells due to limited pilot resource does not vanish with the increased number of BS antennas; hence, PC is recognized as the performance bottleneck of multicell massive MIMO systems [2].

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Much effort has been made to solve the challenging PC problem [7]–[12]. The time-shifted pilot scheme is an effective solution by using asynchronous transmission among adjacent cells [7], but it leads to mutual interferences between data and pilot. A smart pilot assignment scheme is proposed in [8] to optimize the pilot assignment for each cell in a sequential way; however, convergence cannot be guaranteed. PC precoding [9] can mitigate intercell interference by multicell joint processing at the cost of a loss of spectral efficiency due to high overhead required by information exchange. The angle-of-arrival (AOA)-based methods [10], [11] show that geographically separated users with nonoverlapping AOAs do not contaminate each other, even if they adopt the same pilot, but the assumption of small AOA spread of each user is not always true in practical wireless systems. In addition, a blind method based on subspace partitioning [12] is able to reduce intercell interference with heavy computational complexity. Moreover, all those existing solutions ignore the fact that PC severity (PCS) varies among users and try to mitigate PC for all users equally, which leads to a significant efficiency loss since only a part of users suffer from severe PC, while others may enjoy negligible PC.

In this paper, by exploiting different PCSs for different users, a weighted-graph-coloring-based pilot decontamination (WGC-PD) scheme is proposed to mitigate PC for multicell multiuser massive MIMO systems. First, the pilot assignment issue is formulated as a combinational optimization problem to maximize the users' average uplink achievable rate. Then, based on limited cooperation among cells, an edge-weighted interference graph (EWIG) is constructed to depict the potential PC relationship among users, whereby every weighted edge indicates the PC strength introduced between the connected users when they are assigned with the same pilot. Then, by denoting each color as a pilot and each vertex as a user in the EWIG, the proposed WGC-PD scheme, which is developed from the classical DSATUR algorithm [15], greedily assigns different pilots to the connected users with a large weight with insufficient pilot resource. Compared with the exhaustive search to obtain the optimal pilot assignment pattern, the proposed WGC-PD scheme is able to mitigate PC with significantly reduced complexity under the constraint of insufficient pilot resource. Simulation results verify that the proposed WGC-PD scheme outperforms the classical random scheme [1] and the smart pilot assignment scheme [8], and the performance gap to the optimal solutions is also quite small.

II. SYSTEM MODEL

As shown in Fig. 1(a), we consider a multicell multiuser massive MIMO system composed of L hexagonal cells, and each cell consists of a BS with M antennas and K $(K \ll M)$ single-antenna users [1], [2]. The channel vector $\mathbf{h}_{\langle j,k\rangle,i} \in \mathcal{C}^{M\times 1}$ from the kth user in the jth cell to the BS of the ith cell can be modeled as [3]

$$\mathbf{h}_{\langle j,k\rangle,i} = \mathbf{g}_{\langle j,k\rangle,i}\sqrt{\beta_{\langle j,k\rangle,i}} \tag{1}$$

where $\beta_{\langle j,k\rangle,i}$ denotes the large-scale fading coefficient, and $\mathbf{g}_{\langle j,k\rangle,i}$ with distribution $\mathcal{CN}(\mathbf{0},\mathbf{I}_M)$ denotes the small-scale fading vector.

We adopt the widely used block-fading channel model, whereby channel vector $\mathbf{h}_{\langle j,k\rangle,i}$ remains unchanged during the coherence time [1]–[3]. We assume that totally available S $(S \geq K)$ pilots $\boldsymbol{\varphi}_i \in \mathcal{C}^{\tau\times 1}$ $(1 \leq i \leq S)$ of length τ used in one cell are orthogonal to each other, i.e., $\boldsymbol{\Phi} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_S]^T \in \mathcal{C}^{S\times\tau}$, $\boldsymbol{\Phi}\boldsymbol{\Phi}^H = \mathbf{I}_S$, and the same pilot group $\boldsymbol{\Phi}$ is reused in other cells due to limited pilot resource [3], [14]. It should be pointed out that there has been some work discussing pilot reuse technology within one cell by exploiting spatially correlated Rayleigh fading channels [13]. The classical pilot assignment scheme assigns pilot $\boldsymbol{\varphi}_{p_{\langle j,k\rangle}}$ to user $\langle j,k\rangle$ randomly, i.e., $p_{\langle j,k\rangle} \in \{1,2,\ldots,S\}$, and guarantees that no pilots will be reused within one cell, i.e., $p_{\langle j,k\rangle} \neq p_{\langle j,k'\rangle}$ $\forall k \neq k'$ [7]. By adopting a matched-filter receiver at the BS [1], the uplink signal-to-interference-plus-noise ratio (SINR) of user $\langle j,k\rangle$ can be calculated after pilot assignment as

$$\begin{aligned}\mathrm{SINR}^{\mathrm{UL}}_{\langle j,k\rangle} &= \frac{\left\|\mathbf{h}^H_{\langle j,k\rangle,j}\right\|^4}{\sum_{\langle j',k'\rangle\in\mathcal{I}_{\langle j,k\rangle}}\left\|\mathbf{h}^H_{\langle j',k'\rangle,j}\right\|^4 + \sigma^2_{\langle j,k\rangle}/\rho^2}\\ &\approx \frac{\beta^2_{\langle j,k\rangle,j}}{\sum_{\langle j',k'\rangle\in\mathcal{I}_{\langle j,k\rangle}}\beta^2_{\langle j',k'\rangle,j}}, M\to\infty\end{aligned} \tag{2}$$

where $\mathcal{I}_{\langle j,k\rangle} = \{\langle j',k'\rangle : p_{\langle j',k'\rangle} = p_{\langle j,k\rangle}\} \setminus \{\langle j,k\rangle\}$ denotes the set of users with the same pilot as user $\langle j,k\rangle$, $\sigma^2_{\langle j,k\rangle}$ denotes the power of uncorrelated interference and noise that can be substantially reduced by increasing the number of BS antennas M [1], $\sum_{\langle j',k'\rangle\in\mathcal{I}_{\langle j,k\rangle}}\beta^2_{\langle j'k'\rangle,j}$ denotes PC caused by pilot reuse, and ρ denotes the transmit power. Thus, the corresponding average uplink achievable rate of user $\langle j,k\rangle$ can be calculated as

$$C^{\mathrm{UL}}_{\langle j,k\rangle} = (1-\mu_S)E\left\{\log_2(1+\mathrm{SINR}^{\mathrm{UL}}_{\langle j,k\rangle})\right\} \tag{3}$$

where μ_S evaluates the loss of spectral efficiency caused by uplink pilot transmission, which is actually the ratio of pilot length τ and channel coherence time l [2], i.e., $\mu_S = \tau/l$.

From (2) and (3), it is clear that thermal noise and small-scaling fading effects could be averaged out as M grows to infinity, whereas the average uplink achievable rate is limited by PC. For example, as shown in Fig. 1(a), users in the shadow area suffer from severe PC due to the classical random pilot assignment scheme, and the uplink SINR cannot be improved by increasing the number of BS antennas [2].

III. PROPOSED WEIGHTED-GRAPH-COLORING-BASED PILOT DECONTAMINATION SCHEME

A. Problem Formulation

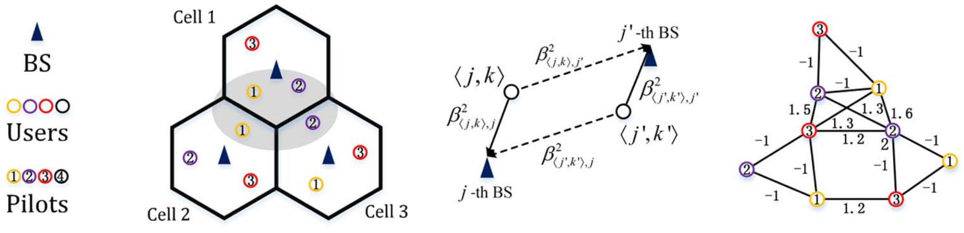
The number of different kinds of pilot assignments for K users and S pilots in the jth cell is $A^K_S = S!/((S-K)!)$. It is apparent that all A^K_S kinds of pilot assignments make no difference when only one cell is considered. However, considering a practical system with L hexagonal cells, the total number of essentially different pilot assignments is as huge as $(A^K_S)^{L-1}$.

In contrast to the classical pilot assignment scheme that allocates pilot $\boldsymbol{\varphi}_{p_{\langle j,k\rangle}}$ to user $\langle j,k\rangle$ randomly [1]–[3], we aim to maximize the total uplink throughput of all KL users in L hexagonal cells, which can be formulated as the following optimization problem P_1:

$$\max_{p_{\langle j,k\rangle}}\left\{\sum_{\langle j,k\rangle}\log_2\left(1+\frac{\left\|\mathbf{h}^H_{\langle j,k\rangle,j}\right\|^4}{\sum_{\langle j',k'\rangle\in\mathcal{I}_{\langle j,k\rangle}}\left\|\mathbf{h}^H_{\langle j',k'\rangle,j}\right\|^4 + \sigma^2_{\langle j,k\rangle}/\rho^2}\right)\right\} \tag{4}$$

where $\{p_{\langle j,k\rangle}\}$ denotes all kinds of pilot assignments, and $\{\langle j,k\rangle\}$ denotes all KL users. However, it is impossible to solve this optimization problem P_1 due to the fact that the BS cannot obtain accurate channel information if PC exists, i.e., $\widehat{\mathbf{h}}_{\langle j,k\rangle,j} = \sum_{p_{\langle j',k'\rangle}=p_{\langle j,k\rangle}}\mathbf{h}_{\langle j',k'\rangle,j}$ [1]. Fortunately, the limit of total uplink throughput can be represented by the large-scale fading coefficients $\beta_{\langle j,k\rangle}$, as shown in (2) and (3). Thus, optimization problem P_1 can be approached by P_2, i.e.,

$$\max_{p_{\langle j,k\rangle}}\left\{\sum_{\langle j,k\rangle}\log_2\left(1+\frac{\beta^2_{\langle j,k\rangle,j}}{\sum_{\langle j',k'\rangle\in\mathcal{I}_{\langle j,k\rangle}}\beta^2_{\langle j',k'\rangle,j}}\right)\right\}. \tag{5}$$



(a) (b) (c)

Fig. 1. EWIG construction. (a) Classical random pilot assignment scheme: The pilots are randomly assigned to the users in each cell, which may cause severe PC among users in adjacent cells, e.g., users in the shadow area. (b) PCS: The ratio between the interference channel strength and the effective channel strength. (c) Construction of the EWIG: An edge-weighted graph can be constructed to describe the potential PC relationship among all users based on the PCS.

Hence, this simplified combinatorial optimization problem P_2 can be solved by exhaustive search among $(A_S^K)^{L-1}$ kinds of pilot assignments. However, for a typical multicell massive MIMO system with $L = 7$ and $S = K = 8$ as an example, the search complexity is $(A_S^K)^{L-1} = (K!)^{L-1} = (8!)^6 \approx 4.3 \times 10^{27}$, which is infeasible in practice.

B. EWIG Construction

By considering two users in different cells with the same pilot, i.e., $\langle j,k\rangle, \langle j',k'\rangle$, $j \neq j'$, $p_{\langle j,k\rangle} = p_{\langle j',k'\rangle}$, we can rewrite the corresponding average uplink achievable rate of user $\langle j,k\rangle$ as

$$C^{\mathrm{UL}}_{\langle j,k\rangle} \propto \log_2\left(1 + \frac{\beta^2_{\langle j,k\rangle,j}}{\beta^2_{\langle j',k'\rangle,j'} + \varepsilon_{j,k,j',k'}}\right) \tag{6}$$

where $\varepsilon_{j,k,j',k'} = \sum_{\langle j_0,k_0\rangle \in \mathcal{I}_{\langle j,k\rangle}, j_0 \neq j'} \beta^2_{\langle j',k'\rangle,j}$ denotes the PC caused by other users with the same pilot, and $C^{\mathrm{UL}}_{\langle j',k'\rangle}$ could be similarly represented. It is clear that different pilot assignments lead to different $\varepsilon_{j,k,j',k'}$ of user $\langle j,k\rangle$, which is difficult to accurately measure. However, the PCs between user $\langle j',k'\rangle$ and user $\langle j,k\rangle$ with the same pilot are closely related to the ratios $\beta^2_{\langle j,k\rangle,j}/\beta^2_{\langle j',k'\rangle,j}$ and $\beta^2_{\langle j',k'\rangle,j'}/\beta^2_{\langle j,k\rangle,j'}$. Hence, we define a metric $\zeta_{\langle j,k\rangle,\langle j',k'\rangle}$ as PCS to measure the strength of potential PC between any two users in different cells, i.e., $\langle j,k\rangle$ and $\langle j',k'\rangle$, $j \neq j'$, which is represented as

$$\zeta_{\langle j,k\rangle,\langle j',k'\rangle} = \beta^2_{\langle j',k'\rangle,j}/\beta^2_{\langle j,k\rangle,j} + \beta^2_{\langle j,k\rangle,j'}/\beta^2_{\langle j',k'\rangle,j'}. \tag{7}$$

Specifically, $\zeta_{\langle j,k\rangle,\langle j',k'\rangle}$ is the ratio between interference channel strength and effective channel strength, and its intuitive illustration is shown in Fig. 1(b). Larger $\zeta_{\langle j,k\rangle,\langle j',k'\rangle}$ indicates that more severe PC will be introduced between user $\langle j,k\rangle$ and user $\langle j',k'\rangle$ when the same pilot is allocated to them. Moreover, it is clear that this PCS metric has a symmetrical property, i.e., $\zeta_{\langle j,k\rangle,\langle j',k'\rangle} = \zeta_{\langle j',k'\rangle,\langle j,k\rangle}$ $\forall j \neq j'$.

Mathematically, the EWIG can be interpreted as an undirected weighted graph $G = (V, E)$, where vertices in set V denote all users, i.e., $V = \{\langle j,k\rangle : 1 \leq j \leq L, 1 \leq k \leq K\}$, and edges in set E denote potential PC among users, i.e., $E = \{\zeta_{\langle j,k\rangle,\langle j',k'\rangle} : j \neq j'\}$. An example of EWIG is shown in Fig. 1(c), where the edges with negligible weights between users in different cells are omitted for simplicity.

C. Proposed WGC-PD Scheme

For two users in different cells in the constructed EWIG, i.e., $\langle j,k\rangle, \langle j',k'\rangle, j' \neq j$, it is clear that the larger PCS $\zeta_{\langle j,k\rangle,\langle j',k'\rangle}$ indicates that more serious PC will be introduced when the same pilot is assigned to them. That is, two users in different cells may reuse the same pilot with negligible performance loss only if their PCS $\zeta_{\langle j,k\rangle,\langle j',k'\rangle}$ is small enough. Thus, we can find that the EWIG is a powerful tool to realize the efficient pilot assignment with significantly reduced PC by greedily assigning different pilots to the connected users with a large weight [16], [17]. The pilot resource is usually limited in practice, e.g., only $K \ll$ KL orthogonal pilots are available in typical multicell massive MIMO systems [2].

To obtain a tradeoff between pilot overhead and the reduction of PC, the WGC-PD scheme is proposed to significantly reduce PC under the constraint of limited pilot resource. Inspired by the classical DSATUR algorithm [15], which sorts vertices according to their degrees in descending order and colors them in a sequential way with reused colors as far as possible, the proposed WGC-PD scheme greedily assigns different pilots to connected users with a large weight in the EWIG. However, unlike the DSATUR algorithm, which ensures that no connected vertices are assigned with the same color, two users in different cells with a small weight in the EWIG may be assigned with the same pilot due to the additional constraint of limited pilot resource in practical massive MIMO systems. Thus, the proposed WGC-PD scheme can be regarded as a specific variant of the classical DSATUR algorithm for edge-weighted graphs under the constraint of insufficient colors.

Mathematically, the pseudocode of the proposed WGC-PD scheme is provided in Algorithm 1, which is mainly comprised of the following three parts.

Algorithm 1 Proposed WGC-PD Scheme

```
Input:
    System Parameters: $K$, $L$, and $S$;
    The constructed EWIG: $G = (V, E)$.
Output:
    Pilot allocation: $\{p_{\langle j,k\rangle}\}$, for $1 \leq j \leq L, 1 \leq k \leq K$.
1:  Initialization:
2:    $\{j_1, k_1, j_2, k_2\} = \arg\max_{\{j,k,j',k',j \neq j'\}} \zeta_{\langle j,k\rangle,\langle j',k'\rangle}$.
3:    $\{p_{\langle j,k\rangle}\} = 0, p_{\langle j_1,k_1\rangle} = 1, p_{\langle j_2,k_2\rangle} = 2$.
4:    $\Omega = \{\langle j_1,k_1\rangle, \langle j_2,k_2\rangle\}$.
5:  while $\exists p_{\langle j,k\rangle} = 0$ do
6:    $\delta_{\langle j,k\rangle} = \sum_{\langle j',k'\rangle \in \Omega, j \neq j'} \zeta_{\langle j,k\rangle,\langle j',k'\rangle}$.
7:    $\langle j_0,k_0\rangle = \arg\max_{\langle j,k\rangle}\{\delta_{\langle j,k\rangle} : \langle j,k\rangle \notin \Omega\}$.
8:    $\Lambda = \{s : \forall k, p_{\langle j_0,k\rangle} \neq s, 1 \leq s \leq S\}$.
9:    $\eta_s = \sum_{\langle j,k\rangle \in \Omega, p_{\langle j,k\rangle} = s} \zeta_{\langle j_0,k_0\rangle,\langle j,k\rangle}$.
10:   $p_{\langle j_0,k_0\rangle} = \arg\min_s\{\eta_s : s \in \Lambda\}$.
11:   $\Omega = \Omega \cap \{\langle j_0,k_0\rangle\}$.
12: end while
13: return $\{p_{\langle j,k\rangle}\}$.
```

1) Initialization (Steps 2–4): First, two users in different cells with the largest weighted edge in the EWIG are selected in step 2, i.e., user $\langle j_1,k_1\rangle$ and user $\langle j_2,k_2\rangle$. In step 3, these two users are assigned with pilots $\boldsymbol{\varphi}_1$ and $\boldsymbol{\varphi}_2$, respectively, and then, they are added to the assigned set Ω as initialization of Ω in step 4. After that, the rest of the users will be selected and assigned with a pilot in a sequential way until all users are assigned with pilots, i.e., $\nexists p_{\langle j,k\rangle} = 0$.

2) User Selection (Steps 6 and 7): To select users in order of importance, a priority parameter $\delta_{\langle j,k\rangle}$ is introduced, which is defined as the weight sum of the edges connecting user $\langle j,k\rangle$ and the users in other cells within Ω in step 6. Then, the user $\langle j_0,k_0\rangle$ with the largest potential PC strength out of the assigned set Ω will be selected in step 7, whose pilot assignment should be preferentially considered.

3) Pilot Assignment (Steps 8–11): After user $\langle j_0,k_0\rangle$ has been selected, the proposed WGC-PD scheme aims to select the pilot causing the smallest potential PC to this user from the available pilot resource. Specifically, the optional pilot set Λ is first constructed in step 8, which includes the pilots unused in the j_0th cell to ensure that no pilots are reused within the same cell, i.e., $p_{\langle j,k\rangle} \neq p_{\langle j,k'\rangle}$, $k \neq k'$. Then, we define η_s in step 9 to describe the potential PC strength between the users with pilot s in Ω and user $\langle j_0,k_0\rangle$ by assuming that user $\langle j_0,k_0\rangle$ is assigned with pilot s. Finally, the pilot having the smallest potential PC strength η_s is selected to be assigned to user $\langle j_0,k_0\rangle$ in step 10, and user $\langle j_0,k_0\rangle$ will be added into the assigned set Ω in step 11. This loop will be carried out in a sequential way until all users are assigned their corresponding pilots.

The proposed WGC-PD scheme is able to mitigate PC under the constraint of limited pilot resource by greedily assigning different pilots to the connected users with a large weight in the EWIG. For instance, as shown in Fig. 1(c), the typical pilot resource, i.e., $S = K = 3$, can be utilized to realize pilot assignment with significantly reduced PC by the proposed WGC-PD scheme.

D. Further Discussion

As indicated in Section III-A, the EWIG is constructed according to the large-scale fading coefficients $\beta_{\langle j,k\rangle,i}$, which denotes the channel strength between users and BSs. Similar to most existing works [8]–[12], which assume that the large-scale fading coefficients are known at BSs, the proposed WGC-PD scheme also requires such prior information. In fact, the large-scale fading coefficients change slowly and can be easily tracked with low complexity in practical mobile systems [18]. For example, in practical mobile cellular networks of the long-term evolution systems [18], users first capture the cell-specific reference signal to measure the channel conditions of available BSs and then select the BS with the best channel condition. Then, users will continue tracking the channel conditions to available BSs to realize the handover process among adjacent cells.

The computational complexity of the exhaustive search solution to combinatorial optimization problems is usually infeasible due to its exponential increasing property [16], [17]. The total computational complexity of the proposed WGC-PD scheme is only $\mathcal{O}(S(\mathrm{KL})^3)$ based on Algorithm 1, which is negligible compared with $\mathcal{O}((A_S^K)^{L-1})$ required by exhaustive search, e.g., $(S(\mathrm{KL})^3)/(A_S^K)^{L-1} = 3.3 \times 10^{-22}$ with $L = 7$ and $S = K = 8$. The time-shifted pilot scheme [7] suffers from the handover complexity of users, since different signal demodulation orders are designed in different cells due to asynchronous transmission. The sequential optimization for each cell in the smart pilot assignment scheme [8] only requires the computational complexity of $\mathcal{O}(LK^2)$, but it leads to poor overall performance. Thus, the proposed WGC-PD scheme is able to achieve an efficient tradeoff between system performance and computational complexity. Apart from maximizing the total uplink rate, we can also formulate optimization problems to maximize the minimum rate among all the users or the sum of minimal rates within each cell. These combinatorial optimization problems could also be solved by similar heuristic methods.

TABLE I
SIMULATION PARAMETERS

Number of cells L	3, 7
Number of BS antennas M	$8 \leq M \leq 256$
Number of users in each cell K	4, 16
Number of orthogonal pilots S	$K \leq S \leq KL$
Cell radius R	500 m
Transmit power ρ	[5, 30] dB
Loss of spectral efficiency $\mu_S = \frac{S}{K}\mu_0$	$\mu_0 = 0.05$
Path loss exponent α	3
Log normal shadowing fading σ_{shadow}	8 dB

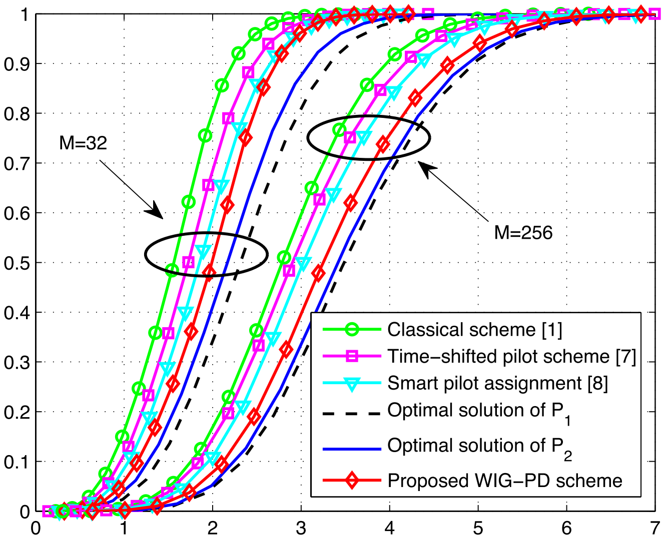


Fig. 2. CDF of the users' uplink achievable rate, where the system parameters $L = 3$, $S = K = 4$, $\rho = 10$ dB, and $M = 32$ ($M = 256$) are considered.

IV. NUMERICAL RESULTS

Here, we investigate the performance of the proposed WGC-PD scheme through Monte Carlo simulations. A typical hexagonal cellular network with L cells is considered, where each cell has K single-antenna users and a BS with M antennas [1], [2]. The spectral efficiency loss with $S = K$ is set as $\mu_0 = 0.05$ [8], and the corresponding μ_S is calculated as $\mu_S = (S/K)\mu_0$. The system parameters are summarized in Table I, e.g., the number of BS antennas ranging from eight to 256 is considered. The locations of users are uniformly randomly distributed in their corresponding cells in each trial. As addressed in [2], the large-scale fading coefficient $\beta_{\langle j,k\rangle,i}$ can be modeled as

$$\beta_{\langle j,k\rangle,i} = \frac{z_{\langle j,k\rangle,i}}{\left(\frac{r_{\langle j,k\rangle,i}}{R}\right)^{\alpha}} \tag{8}$$

where $z_{\langle j,k\rangle,i}$ represents shadow fading and typically possesses a lognormal distribution (i.e., $10\log_{10}(z_{\langle j,k\rangle,i})$ is Gaussian distributed with zero mean and a standard deviation of σ_{shadow}), $r_{\langle j,k\rangle,i}$ is the distance between the kth user in the jth cell and the BS in the ith cell, and R is the cell radius.

Fig. 2 shows the cumulative distribution function (cdf) curve of the users' uplink achievable rate, where the system parameters $L = 3$, $S = K = 4$, $\rho = 10$ dB, and $M = 32$ ($M = 256$) are considered. The proposed WGC-PD scheme is compared with the following existing solutions: The classical pilot assignment scheme randomly assigns pilots to users without cooperation among cells [1]; the time-shifted pilot scheme cancels PC by using asynchronous transmission among adjacent cells at the cost of introducing mutual interferences between

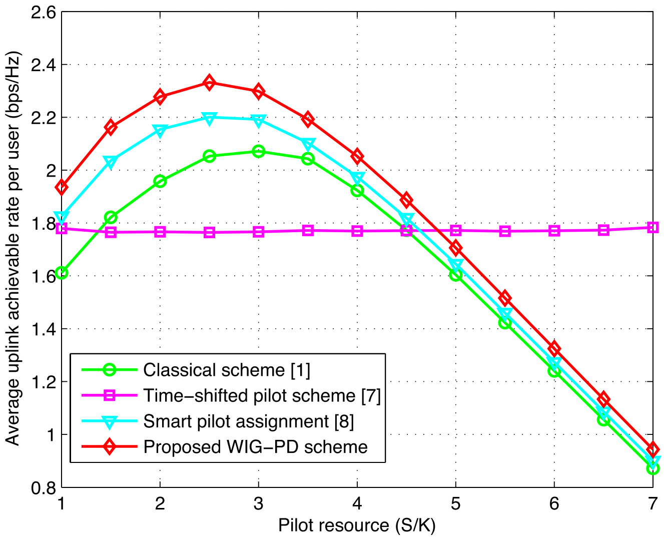


Fig. 3. Average uplink achievable rate per user against the number of pilot resource S, where the system parameters $L = 7$, $K = 16$, $\rho = 10$ dB, and $M = 128$ are considered.

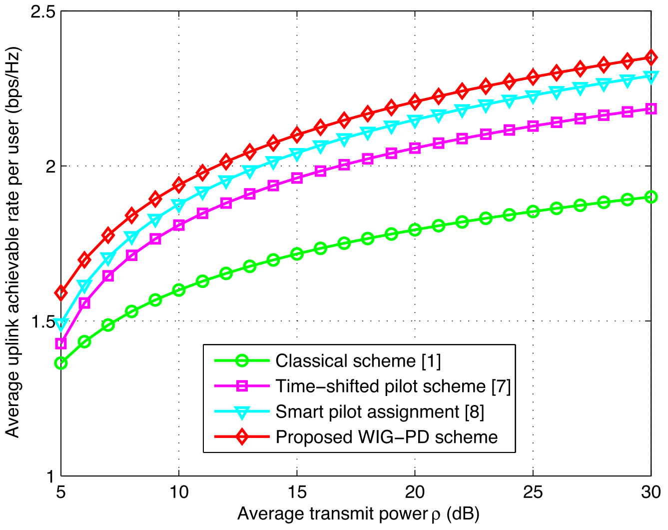


Fig. 4. Average uplink achievable rate per user against the average signal-to-noise ratio γ at the BS, where the system parameters $L = 7$, $K = 16$, $S = 16$, and $M = 128$ are considered.

data and pilots [7]; the smart pilot assignment scheme optimizes pilot assignment for each cell in a sequential way [8]; the optimal solutions to P_1 and P_2 are obtained through exhaustive search. As mentioned in Section III-C, the optimal solution to P_2 is able to approach the optimal solution to P_1 when the BS is equipped with a large number of antennas, which is verified in Fig. 2 with two cases as $M = 32$ and $M = 256$. Moreover, we can find that the proposed WGC-PD scheme outperforms the time-shifted pilot scheme and the smart pilot assignment scheme, and the performance gap between the proposed scheme and the optimal solution to P_1 is about 0.3 b/s/Hz.

Fig. 3 shows the average uplink achievable rate per user against the number of pilot resource S, where the system parameters $L = 7$, $K = 16$, $\rho = 10$ dB, and $M = 128$ are considered. For the time-shifted pilot scheme [7], having more pilots, i.e., $S > K$, makes no performance improvement, since adjacent cells asynchronously transmit uplink pilots. Hence, we assume that fixed $S = K = 16$ pilots are utilized in the time-shifted pilot scheme. Considering pilot resource $1 \leq S/K \leq 4$, it is clear that the proposed WGC-PD scheme significantly outperforms the time-shifted pilot scheme. For the classical random pilot assignment scheme [1], the smart pilot assignment scheme [8], and the proposed WGC-PD scheme, the performance can be improved by increasing the pilot resource when $S/K \leq 3$. However, by continually increasing pilot resource, their performances drop fast due to the increasing loss of spectral efficiency, i.e., $\mu_S = (S/K)\mu_0$.

Fig. 4 shows the average uplink achievable rate per user against the transmit power ρ at users, where the system parameters $L=7$, $K=16$, $S = 16$, and $M = 128$ are considered. It is clear that all considered schemes are able to improve the average uplink achievable rate when ρ is increased. When the average uplink achievable rate is equal to 2 b/s/Hz, the proposed WGC-PD scheme outperforms the time-shifted pilot scheme and the smart pilot assignment scheme by about 2 and 4 dB, respectively.

V. CONCLUSION

In this paper, we have proposed a pilot assignment scheme based on weighted graph coloring to mitigate PC for multicell massive MIMO systems. First, an EWIG is constructed to depict the potential PC relationship among all users. Then, inspired by classical graph coloring algorithms from graph theory, the WGC-PD scheme is proposed to mitigate PC by greedily assigning different pilots to connected users with a large weight in the EWIG. Simulation results demonstrate that the proposed WGC-PD scheme outperforms the existing schemes by about 0.2 b/s/Hz when a typical number of BS antennas $M = 128$ is considered in a multicell massive MIMO system.

REFERENCES

[1] T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[2] F. Rusek *et al.*, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.

[3] B. Lee and B. Shim, "A vector perturbation with user selection for multiuser MIMO downlink," *IEEE Trans. Commun.*, vol. 60, no. 11, pp. 3322–3331, Nov. 2012.

[4] Z. Gao, C. Zhang, and Z. Wang, "Robust preamble design for synchronization, signaling transmission, and channel estimation," *IEEE Trans. Broadcast.*, vol. 61, no. 1, pp. 98–104, Mar. 2015.

[5] D. Ciuonzo, P. Salvo Rossi, and S. Dey, "Massive MIMO channel-aware decision fusion," *IEEE Trans. Signal Process.*, vol. 63, no. 3, pp. 604–619, Feb. 2015.

[6] F. Jiang, J. Chen, A. Swindlehurst, and J. Lopez-Salcedo, "Massive MIMO for wireless sensing with a coherent multiple access channel," *IEEE Trans. Signal Process.*, vol. 63, no. 12, pp. 3005–3017, Jun. 2015.

[7] F. Fernandes, A. Ashikhmin, and T. Marzetta, "Inter-cell interference in noncooperative TDD large scale antenna systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 192–201, Feb. 2013.

[8] X. Zhu, Z. Wang, L. Dai, and C. Qian, "Smart pilot assignment for massive MIMO," *IEEE Commun. Lett.*, vol. 19, no. 9, pp. 1644–1647, Sep. 2015.

[9] A. Ashikhmin and T. Marzetta, "Pilot contamination precoding in multi-cell large scale antenna systems," in *Proc. IEEE ISIT*, Jul. 2012, pp. 1137–1141.

[10] H. Yin, D. Gesbert, M. Filippou, and Y. Liu, "A coordinated approach to channel estimation in large-scale multiple-antenna systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 264–273, Feb. 2013.

[11] H. Yin, L. Cottatellucci, D. Gesbert, R. Muller, and G. He, "Pilot decontamination using combined angular and amplitude based projections in massive MIMO systems," *IEEE Int. SPAWC*, Jun. 2015, pp. 216–220.

[12] R. Muller, L. Cottatellucci, and M. Vehkapera, "Blind pilot decontamination," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 773–786, Oct. 2014.

[13] L. You, X. Gao, X. Xia, N. Ma, and Y. Peng, "Pilot reuse for massive MIMO transmission over spatially correlated Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3352–3366, Jun. 2015.

[14] L. Dai, J. Wang, Z. Wang, P. Tsiaflakis, and M. Moonen, "Time domain synchronous OFDM based on simultaneous multi-channel reconstruction," in *Proc. IEEE ICC*, Budapest, Hungary, Jun. 2013, pp. 2984–2989.

[15] V. Doshi, D. Shah, M. Medard, and M. Effros, "Functional compression through graph coloring," *IEEE Trans. Inf. Theory*, vol. 56, no. 8, pp. 3901–3917, Aug. 2010.

[16] W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations*. North Chelmsford, MA, USA: Courier, 2004.

[17] M. Aigner, *Combinatorial Theory*. Berlin, Germany: Springer, 2012.

[18] S. Sesia, I. Toufik, and M. Baker, *LTE-The UMTS Long Term Evolution: From Theory to Practice*. New York, NY, USA: Wiley, 2009.

A Scalable Performance–Complexity Tradeoff for Constellation Randomization in Spatial Modulation

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***Abstract*—It is widely recognized that traditional single radio frequency (RF)-chain-aided spatial modulation (SM) does not offer any transmit diversity gain. As a remedy, constellation randomization (CR), relying on transmit prescaling (TPS), has been shown to provide transmit diversity for single-RF-chain-aided SM. In this paper, we propose a low-complexity approach to SM with the aid of constellation randomization (SM-CR) that considerably improves the transmit diversity gain of SM at a reduced computational burden compared with conventional SM-CR. While conventional SM-CR performs a full search among a set of candidate TPS factors to achieve the maximum minimum Euclidean distance (MED) in the received SM constellation, here, we propose a thresholding approach, where, instead of the maximum MED, the TPS aims to satisfy a specific MED threshold. This technique offers a significant complexity reduction with respect to the full maximization of SM-CR, since the search for TPS is terminated once a TPS set is found that satisfies the MED threshold. Our analysis and results demonstrate that a scalable tradeoff can be achieved between transmit diversity and complexity by appropriately selecting the MED threshold, where a significant complexity reduction is attained, while achieving a beneficial transmit diversity gain for the single-RF SM.**

***Index Terms*—Constellation shaping, multiple-input single-output, spatial modulation (SM), transmit prescaling (TPS).**

I. INTRODUCTION

Spatial modulation (SM) has been shown to offer a low-complexity design alternative to spatial multiplexing, where only a subset (down to one) of radio frequency (RF) chains is required for transmission [1], [2]. Early work has focused on the design of receiver algorithms for minimizing the bit error ratio of SM at low complexity [1]–[5]. Matched filtering is shown to be a low-complexity technique for detecting the activated antenna index (AI) [1]–[3]. A maximum likelihood (ML) detector is introduced in [4] for reducing the complexity of classic spatial multiplexing ML detectors, whereas the complexity imposed can be further reduced by compressive sensing detection approaches [5]. In addition to receive processing, several transmit precoding (TPC) approaches have been proposed for receive antenna (RA)-aided SM, where the spatial information is mapped onto the RA index [6]–[8].

Relevant work has also proposed constellation shaping for SM [9]–[14]. Specifically, in [9], the transmit diversity of coded SM is analyzed for different *spatial constellations*, which represent the legitimate sets of activated transmit antennas (TAs). Furthermore, Yang *et al.* in [10] conceived a symbol constellation optimization technique for minimizing the bit error rate (BER). Indeed, spatial and symbol constellation shaping are discussed separately in the aforementioned reference. By contrast, the design of the received SM constellation that combines the choice of the TA as well as the transmit symbol constellation is the focus of this paper. A number of constellation shaping schemes [11]–[14] have also been proposed for the special case of SM, which is referred to as space shift keying, where the information is purely carried in the spatial domain, by the activated AI. However, the application of the above constellation shaping to the SM transmission, where the transmit waveform is modulated, is nontrivial.

Recent work has focused on shaping the receive SM constellation by means of symbol prescaling at the transmitter, aiming for maximizing the minimum Euclidean distance (MED) in the received SM constellation [15]–[17]. The constellation shaping approach in [15] and [16] aims at fitting the receive SM constellation to one of the existing optimal classic constellation formats in terms of minimum distance, such as, e.g., quadrature amplitude modulation (QAM). Due to the strict constellation fitting requirement imposed on both amplitude and phase, this prescaling relies on the inversion of the channel coefficients. In the case of ill-conditioned channels, this substantially reduces the received signal-to-noise ratio (SNR). This problem has been alleviated in [17], where a constellation shaping scheme based on phase-only scaling is proposed. Still, the constellation shaping used in the above schemes is limited in the sense that it only applies to multiple-input–single-output systems, where a single symbol is received for each transmission, and thus, the characterization and shaping of the receive SM constellation is simple.

Closely related to this work, a transmit prescaling (TPS) scheme was proposed for SM [19], where the received SM constellation is randomized by TPS for maximizing the MED between its points for a given channel. A number of randomly generated candidate sets of TPS factors are formed offline, known to both the transmitter and the receiver, and the transmitter then selects that particular set of TPS factors that yields the SM constellation having the maximum MED. Against this background, in this paper, we propose a low-complexity relaxation of the above optimization instead of an exhaustive search, where the first TPS factor set that is found to satisfy a predetermined threshold is selected, thus reducing the computational burden of the TPS operation. The proposed scheme is shown to provide a scalable tradeoff between the performance attained and the complexity imposed, by accordingly selecting the MED threshold.

This paper is organized as follows: In Section II, the basic system model is first introduced, and the proposed scheme is then discussed. The computational complexity of the proposed technique is analyzed in Section III, and its performance against the state of the art is evaluated in Section IV. Finally, in Section V, we draw the key conclusions of our study.

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