Effective Rate Analysis of MISO Systems over α-μ Fading Channels

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Outline

- Effective rate is an appropriate metric to quantify the system performance under QoS limitations and is given by [1]

$$\alpha(\theta) = -(1/\theta T)\ln\left(\mathbb{E}\left\{\exp\left(-\theta TC\right)\right\}\right), \quad \theta \neq 0 \tag{1}$$

where C is the system throughput, T denotes the block duration and θ is the QoS exponent.

- For $\theta \to 0$, the Effective Rate reduces to the standard ergodic capacity.

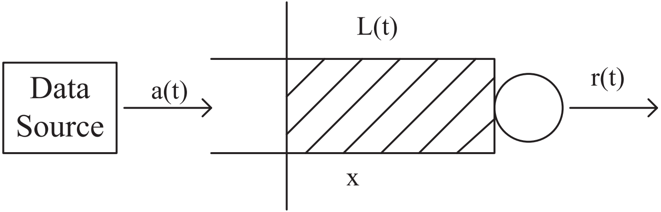


Figure 1: Queuing model

The α-μ distribution provides better fit to experimental data than most existing fading models and involves as special cases:

- Rayleigh
- One-sided Gaussian
- Nakagami-m
- Weibull
- Exponential
- Gamma

The power PDF of α-μ variables is given by [2]

$$f_{\gamma_1}(\gamma_1) = \frac{\alpha_1 {\gamma_1}^{\alpha_1\mu_1/2-1}}{2\beta_1^{\alpha_1\mu_1/2}\Gamma(\mu_1)} \exp\left(-\left(\frac{\gamma_1}{\beta_1}\right)^{\alpha_1/2}\right), \tag{2}$$

where $\beta_1 \triangleq \mathrm{E}\{\gamma_1\}\frac{\Gamma(\mu_1)}{\Gamma\left(\mu_1+\frac{2}{\alpha_1}\right)}$ with $\mathrm{E}\{\gamma_1\} = \hat{r}_1^2 \frac{\Gamma\left(\mu_1+\frac{2}{\alpha_1}\right)}{\left(\mu_1^{\frac{2}{\alpha_1}}\Gamma(\mu_1)\right)}$, and $\hat{r}_1$ is defined as the α_1-root mean value of the envelope random variable R, i.e., $\hat{r}_1 = \sqrt[\alpha_1]{E\{R^{\alpha_1}\}}$.

We consider a MISO system:

$$y = \mathbf{h}\mathbf{x} + n \tag{3}$$

where $\mathbf{h} \in \mathbb{C}^{1\times N_t}$ denotes the channel fading vector, $\mathbf{x}$ is the transmit vector with covariance $\mathtt{E}\{\mathbf{x}\mathbf{x}^\dagger\} = \mathbf{Q}$, and n represents the AWGN term. The effective rate of the MISO channel can be expressed as [3]

$$R(\rho, \theta) = -\frac{1}{A}\log_2\left(\mathtt{E}\left\{\left(1 + \frac{\rho}{N_t}\mathbf{h}\mathbf{h}^\dagger\right)^{-A}\right\}\right) \ \text{bits/s/Hz} \tag{4}$$

where $A = \frac{\theta T B}{\ln 2}$, with B denoting the bandwidth of the system, while ρ is the average transmit SNR.

Exact Effective Rate I

Lemma 1

[4] The sum of i.i.d. squared α-μ RVs with parameters α_1, μ_1, and $\hat{r}_1$, i.e., $\gamma = \sum_{k=1}^{N_t} \gamma_i$, can be approximated by an α-μ RV with parameters α, μ and $\hat{r}$ by solving the following nonlinear equations

$$\frac{\mathrm{E}^2(\gamma)}{\mathrm{E}(\gamma^2) - \mathrm{E}^2(\gamma)} = \frac{\Gamma^2(\mu + 1/\alpha)}{\Gamma(\mu)\Gamma(\mu + 2/\alpha) - \Gamma^2(\mu + 1/\alpha)},$$
$$\frac{\mathrm{E}^2(\gamma^2)}{\mathrm{E}(\gamma^4) - \mathrm{E}^2(\gamma^2)} = \frac{\Gamma^2(\mu + 2/\alpha)}{\Gamma(\mu)\Gamma(\mu + 4/\alpha) - \Gamma^2(\mu + 2/\alpha)},$$
$$\hat{r} = \frac{\mu^{1/\alpha}\Gamma(\mu)\mathrm{E}(\gamma)}{\Gamma(\mu + 1/\alpha)}, \tag{5}$$

As such, we can easily obtain the sum PDF as

$$f_\gamma(\gamma) \approx \frac{\alpha\gamma^{\alpha\mu/2-1}}{2\beta^{\alpha\mu/2}\Gamma(\mu)} \exp\left(-\left(\frac{\gamma}{\beta}\right)^{\alpha/2}\right). \tag{6}$$

Exact Effective Rate II

Proposition 1

For MISO α-μ fading channels, the effective rate is given by

$$R(\rho,\theta) = \frac{1}{A} - \frac{1}{A}\log_2\left(\frac{\alpha\sqrt{k}\,l^{A-1}(N_t\beta/\rho)^{\alpha\mu/2}}{(2\pi)^{l+k/2-3/2}\Gamma(A)\Gamma(\alpha)}\right)$$
$$-\frac{1}{A}\log_2\left(G_{l,k+l}^{k+l,l}\left[\frac{(N_t/\rho)^l}{\left(\beta^{\alpha/2}k\right)^k}\,\middle|\, \begin{matrix}\Delta(l,1-\alpha\mu/2)\\ \Delta(k,0),\Delta(l,A-\alpha\mu/2)\end{matrix}\right]\right), \quad (7)$$

where $G(\cdot)$ is the Meijer's G-function, $\Delta(\epsilon,\tau) = \frac{\tau}{\epsilon}, \frac{\tau+1}{\epsilon}, \cdots, \frac{\tau+\epsilon-1}{\epsilon}$, with τ being an arbitrary real value and ϵ a positive integer. Moreover, $l/k = \alpha/2$, where l and k are both positive integers.

For large values of l and k, it is not very efficient to compute (7).

Exact Effective Rate III

Proposition 2

The effective rate in (7) can be further written in the form of Fox's H-functions by using the Mellin–Barnes integral as

$$R(\rho,\theta) = \frac{1}{A}\left(1 - \log_2\left(\frac{\alpha}{\Gamma(A)\Gamma(\mu)}\right) - \log_2\left(H_{1,2}^{2,1}\left[\left(\frac{N_t}{\rho\beta}\right)^{\alpha/2} \middle| \begin{array}{c}(1,\alpha/2)\\ (\mu,1),(A,\alpha/2)\end{array}\right]\right)\right). \tag{8}$$

It is worth to mention that (8) is very compact which simplifies the mathematical algebraic manipulations encountered in the effective rate analysis.

High-SNR Effective Rate

Proposition 3

For MISO α-μ fading channels, the effective rate at high SNRs is given by

$$R^{\infty}(\rho,\theta) \approx \log_2\left(\frac{\beta\rho}{N_t}\right) - \frac{1}{A}\log_2\left(\frac{\Gamma\left(\mu - 2A/\alpha\right)}{\Gamma\left(\mu\right)}\right). \quad (9)$$

The above result indicates that the high-SNR slope is $S_{\infty} = 1$, which is independent of β. The same observations were made in previous works for the Rayleigh, Rician, and Nakagami-m cases.

Low-SNR Effective Rate

Proposition 4

For MISO α-μ fading channels, the effective rate at low SNRs is given by

$$R\left(\frac{E_b}{N_0},\theta\right) \approx S_0 \log_2\left(\frac{E_b}{N_0} \Big/ \frac{E_b}{N_0}_{\min}\right), \tag{10}$$

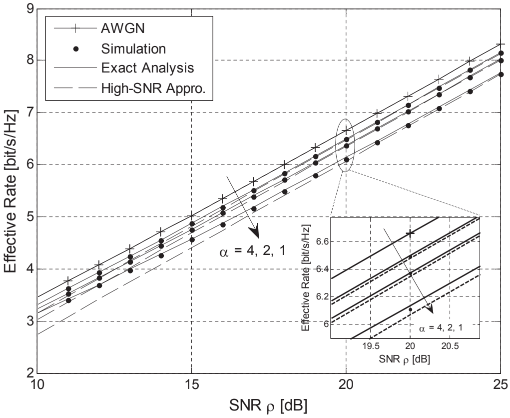
where

$$\frac{E_b}{N_0}_{\min} = \frac{\Gamma\left(\mu_1\right)\ln 2}{\beta_1 \Gamma\left(\mu_1 + 2/\alpha_1\right)}, \tag{11}$$

$$S_0 = \frac{2N_t\Gamma^2\left(\mu+2/\alpha\right)}{\left(A+1\right)\left(\Gamma\left(\mu+4/\alpha\right)\Gamma\left(\mu\right) - \Gamma^2\left(\mu+2/\alpha\right)\right) + N_t\Gamma^2\left(\mu+2/\alpha\right)}. \tag{12}$$

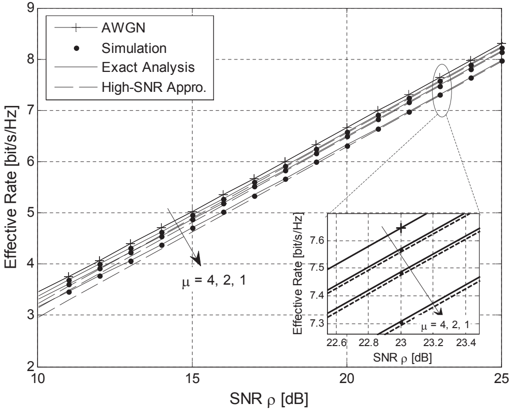
The minimum $\frac{E_b}{N_0}$ is independent of the delay constraint A, whereas the wideband slope S_0 is independent of β, and a decreasing function in A, while it is a monotonically increasing function in N_t.

Numerical Results I



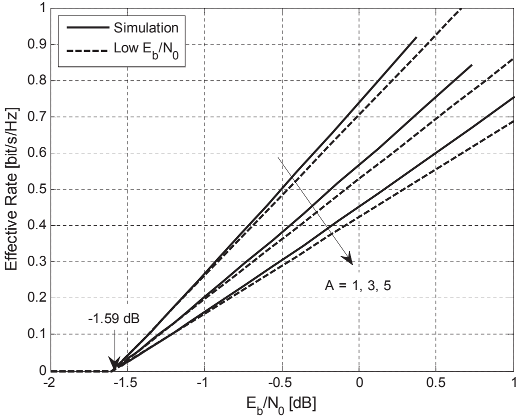
- The exact analytical expression is very accurate for all SNRs,
- The high-SNR approximation is quite tight even in moderate SNRs and its accuracy is improved for larger values of the fading parameters,
- An increase of the effective rate is observed as α increases.

Numerical Results II



- An increase of the effective rate is observed as α increases,
- Since a large value of μ results in more multipath components,
- A large value of α accounts for a larger fading gain,

Numerical Results III



- The effective rate is a monotonically decreasing function of A, which implies that tightening the delay constraints reduces the effective rate,
- The change of the delay constraint A does not affect the minimum E_b/N_0, which is -1.59 dB in our case,

Conclusions

- Novel and analytical expressions of the exact effective rate of MISO systems over i.i.d. α-μ fading channels have been derived by using an α-μ approximation.
- From high-SNR approximation, the effective rate can be improved by utilizing more transmit antennas as well as in a propagation environment with larger values of α and μ.
- Our analysis provides the minimum required transmit energy per information bit for reliably conveying any non-zero rate at low SNRs.
- Our analytical results serve as a performance benchmark for our future work on the performance analysis of the multi-user scenario.

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Thank you!