Performance Analysis of Relay Assisted Cooperative Non-Orthogonal Multiple Access Systems

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Abstract—Non-orthogonal multiple access (NOMA) is a promising multiple access technique for the fifth generation (5G) wireless communications. In order to enhance the performance gains of NOMA systems, a relay assisted cooperative NOMA scheme is designed in this paper. In the proposed scheme, the concept of NOMA is exploited to realize the transmission of the source information in the second time slot. The destination can only receives a single data symbol in conventional cooperative systems, while the destination can reliably acquire two data symbols in two time slots. Therefore, the proposed scheme can achieve higher achievable rate performance than existing schemes. Moreover, the complex power allocation algorithm can be avoided to alleviate the complexity of cooperative NOMA systems. For Rayleigh fading channels, we derive exact expressions for two important performance metrics of the proposed scheme, i.e., the outage probability and the achievable rate. In addition, asymptotic results are presented in terms of simple elementary functions to provide useful insights to guide the practical implementation. Monte-Carlo simulations are provided to demonstrate the performance of the proposed scheme and the accuracy of the derived analytical results.

Index Terms—Non-orthogonal multiple access, cooperative relay, outage probability, achievable rate.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) is one of the most promising multiple access techniques to support massive connectivity in future fifth generation (5G) wireless networks [1], [2]. Compared to conventional orthogonal multiple access (OMA) techniques, NOMA can serve multiple users using the same resources in the time/frequency/code domain but with different power levels, i.e., multiple users' symbols are multiplexed using superposition coding in the power domain [1]. At the NOMA receiver, we can first decode the symbol with the best quality, and then apply successive interference cancellation (SIC) [3] to detect the remaining symbols.

On the other hand, cooperative relay transmission has been shown to be able to significantly improve the transmission reliability, network coverage, and achievable rate of cellular networks [4], [5]. Thus, the integration of NOMA and cooperative relaying has recently attracted increasing interests to improve the throughput of future 5G wireless networks [6], [7]. Specifically, a cooperative NOMA transmission scheme was proposed in [8], where the users with better channel qualities can be severed as relays to improve the performance of the users with poor channel qualities. More recently, NOMA with a dedicated relay was proposed to improve the transmission reliability for a user with poor channel qualities [9], while the authors in [10] presented the outage behavior analysis of NOMA with multiple-antenna relays. Furthermore, a cooperative relaying systems using NOMA (CRS-NOMA) was proposed in [11], where only the relay transmits the decoded symbol to the destination in the second time slot. However, most of existing schemes have not considered to send the source symbols in the second time slot, which fails to fully exploit the NOMA principle for further performance enhancement. In addition, power allocation with high complexity is required by these schemes to realize high achievable rates.

In this paper, we propose a relay assisted cooperative NOMA scheme to fully exploit the NOMA principle to enhance the performance gain further. Specifically, in the first slot, the source transmits the same symbol to the destination and the relay, respectively [1]. In the second time slot, unlike the existing CRS-NOMA scheme [11], the source in the proposed scheme can transmit symbols to the destination, and the destination can use the SIC detection strategy to reliably decode the symbols. Therefore, the proposed scheme can achieve higher achievable rates than existing CRS-NOMA [11] and CRS-OMA [4] schemes. Moreover, because each node transmits the data symbol with its maximum power, the complicated power allocation required by previous works is no longer needed by the proposed scheme. Finally, the exact expression for the outage probability as well as its high-SNR approximation are derived to show that the diversity order of the proposed scheme is the same as existing CRS-NOMA schemes.

The rest of the paper is organized as follows: Section II introduces the system model of the proposed relay assisted cooperative NOMA. In Section III, we provide a detailed analysis for the outage probability and achievable rate of the proposed scheme. Numerical results are provided in Section IV to validate the theoretical analysis. Finally, Section V concludes this paper by summarizing the key findings.

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[1]Simulation codes are provided to reproduce the results presented in this paper: http://oa.ee.tsinghua.edu.cn/dailinglong/.

(a) CRS-NOMA.

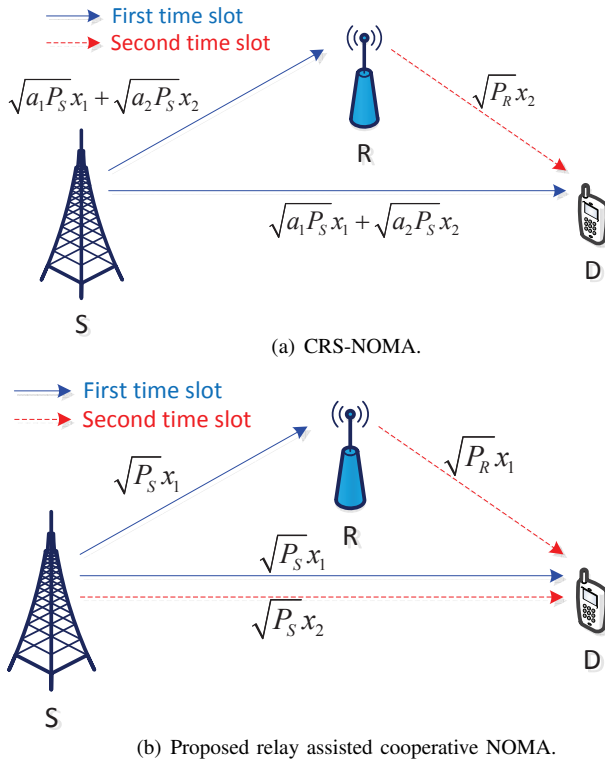(b) Proposed relay assisted cooperative NOMA.

Fig. 1.   Two system models of different relaying protocols.

II. SYSTEM MODEL OF THE PROPOSED SCHEME

As illustrated in Fig. 1(b), we consider a simple cooperative network where a source node, S, communicates with a destination node, D, with the assistance of a fixed decode-and-forward (DF) [4] relay node, R, which fully decodes, re-encodes, and retransmits the source message to the destination D. Moreover, the source S can also directly communicate the destination D. We further assume the relay works in the half-duplex mode, i.e., the relay cannot transmit and receive a symbol at the same time. The general notation h_a, $a \in \{SR, SD, RD\}$, is defined to denote the channel coefficients of the link between S, R and D, respectively. Without loss of generality, the fading gains in all involved links are assumed to follow the Rayleigh distribution with the probability density function (PDF) given by

$$f_{h_a}(x) = \frac{1}{\Omega_a} \exp\left(-\frac{x}{\Omega_a}\right), \quad a \in \{SR, SD, RD\}, \quad (1)$$

where Ω_a denotes the average power, and the cumulative distribution function (CDF) given by

$$F_{h_a}(x) = 1 - \exp\left(-\frac{x}{\Omega_a}\right), \quad a \in \{SR, SD, RD\}. \quad (2)$$

The communication process for cooperative relay systems consists of two consecutive time slots. During the first time slot, the source S transmits a symbol x_1 with the power P_S to both the relay R and the destination D. Accordingly, the received signals at R and D can be expressed as

$$r_{a,x_1} = \sqrt{P_S} h_a x_1 + n, \quad a \in \{SR, SD\}, \quad (3)$$

where n is the additive white Gaussian noise (AWGN) with zero mean and variance σ^2. We assume that $E\{|x_1|^2\} = 1$ with $E\{\cdot\}$ denoting expectation. The received SNRs for symbol x_1 at R and D are given by

$$\gamma_{a,x_1} = \frac{P_S |h_a|^2}{\sigma^2} = \rho |h_a|^2, \quad a \in \{SR, SD\}, \quad (4)$$

where $\rho \triangleq P_S/\sigma^2$ represents the transmit SNR.

In the second time slot, R forwards the decoded symbol x_1 with the power P_R to D, and S transmits another symbol x_2 with $E\{|x_2|^2\} = 1$ and the power P_S to D. While in the CRS-NOMA scheme [11], only the relay transmits the decoded symbol to the destination D as in Fig. 1(a). Then, the received signal at D is given by

$$r_D = \sqrt{P_R} h_{RD} x_1 + \sqrt{P_S} h_{SD} x_2 + n. \quad (5)$$

Usually, the fading gain h_{SD} of the S-D link is smaller than the fading gain h_{RD} of the R-D link, i.e., $E\{|h_{SD}|^2\} < E\{|h_{RD}|^2\}$, due to the distance difference. This natural characteristic of the channel difference between different transceivers facilitates us to utilize the NOMA principle at the second time slot. However, CRS-NOMA in [11] employs complex power allocation at the source to distinguish two symbols, i.e., power allocation coefficients a_1 and a_2. According to the SIC-based NOMA scheme, the destination D firstly decodes the symbol x_1 by treating the symbol x_2 as a noise term. Then, x_1 is canceled from r_D by using SIC to decode x_2. Since we are interested in the effect of channel gains on the outage performance, we follow the typical assumption [11] that the transmit power of S and R is the same as $P_S = P_R = P$. Therefore, the received SNRs for symbols x_1 and x_2 are respectively obtained as

$$\gamma_{D,x_1} = \frac{\rho |h_{RD}|^2}{\rho |h_{SD}|^2 + 1}, \quad (6)$$

$$\gamma_{D,x_2} = \rho |h_{SD}|^2. \quad (7)$$

Since we utilize the fixed DF scheme at the relay, the end-to-end SNR for the transmitted symbol x_1 with the aid of R can be expressed as

$$\gamma_{e2e,x_1} = \min\{\gamma_{D,x_1}, \gamma_{SR,x_1}\}. \quad (8)$$

III. PERFORMANCE ANALYSIS

In this section, we derive analytical expressions for the outage probability and achievable rate of the proposed relay assisted cooperative NOMA system in Section II. In addition, asymptotic high-SNR expressions are also presented to provide useful insights into the effects of system and channel parameters on the performance. Finally, our derived result will be compared with existing results.

A. Outage Probability

We first illustrate the outage probability of the relay assisted cooperative NOMA system. The outage probability is defined as the probability that the instantaneous received SNR γ falls below a pre-defined threshold γ_{th}. Thus, we can use the CDF

of the received SNR to readily evaluate the outage probability as shown in the following Theorem 1.

Theorem 1. *For the relay assisted cooperative NOMA system, the outage probabilities P_{o,x_1} and P_{o,x_2} for symbols x_1 and x_2 are respectively given by*

$$P_{o,x_1} = 1 - \exp\left(-\frac{\gamma_{th}}{\rho\Omega_{SD}}\right) - \frac{\Omega_{RD} e^{-\frac{\gamma_{th}}{\rho}\left(\frac{1}{\Omega_{RD}} + \frac{1}{\Omega_{SR}}\right)}}{\gamma_{th}\Omega_{SD} + \Omega_{RD}}$$
$$+ \frac{\Omega_{RD} e^{-\frac{\gamma_{th}}{\rho}\left(\frac{1}{\Omega_{RD}} + \frac{1}{\Omega_{SR}} + \frac{1}{\Omega_{SD}}\right)}}{\gamma_{th}\Omega_{SD} + \Omega_{RD}}, \tag{9}$$

$$P_{o,x_2} = 1 - \frac{\Omega_{RD}}{\gamma_{th}\Omega_{SD} + \Omega_{RD}} \exp\left(-\frac{\gamma_{th}}{\rho\Omega_{SD}} - \frac{\gamma_{th}}{\rho\Omega_{RD}}\right). \tag{10}$$

Proof: Employing the definition of outage probability

$$P_o = \Pr\left(\gamma < \gamma_{th}\right) = F_\gamma\left(\gamma_{th}\right), \tag{11}$$

we can directly derive the outage probability of x_1 in the S-D link, P_{o,x_1}, as

$$P_{SD,x_1} = 1 - \exp\left(-\frac{\gamma_{th}}{\rho\Omega_{SD}}\right). \tag{12}$$

To derive the outage probability of x_1, the CDF of γ_{e2e} should be obtained at first. With the help of (1), (2) and (6), the CDF of γ_{D,x_1} is given by [12, Eq.(2)]

$$F_{D,x_1}(z) = \int_0^\infty F_X\left(z\left(y+1\right)\right) f_Y\left(y\right) dy$$
$$= 1 - \int_0^\infty \frac{1}{\rho\Omega_{SD}} \exp\left(-\frac{z\left(y+1\right)}{\rho\Omega_{RD}} - \frac{y}{\Omega_{SD}}\right) dy$$
$$= 1 - \frac{\Omega_{RD}}{z\Omega_{SD} + \Omega_{RD}} \exp\left(-\frac{z}{\rho\Omega_{RD}}\right), \tag{13}$$

where $X \triangleq \rho|h_{RD}|^2$, $Y \triangleq \rho|h_{SD}|^2$, and $Z \triangleq \gamma_{D,x_1}$. In fixed DF relaying systems, an outage event occurs if either one of the two-hop links cannot decode x_1. Using (13), the outage probability can be expressed as

$$P_{e2e,x_1}\left(\gamma_{th}\right) \triangleq F_{C,x_1}\left(\gamma_{th}\right)$$
$$\triangleq F_{D,x_1}\left(\gamma_{th}\right) + F_{SD,x_1}\left(\gamma_{th}\right) - F_{D,x_1}\left(\gamma_{th}\right) F_{SD,x_1}\left(\gamma_{th}\right)$$
$$= 1 - \frac{\Omega_{RD} e^{-\frac{\gamma_{th}}{\rho}\left(\frac{1}{\Omega_{RD}} + \frac{1}{\Omega_{SR}}\right)}}{\gamma_{th}\Omega_{SD} + \Omega_{RD}}. \tag{14}$$

The selection combining technique is used at the receiver, and the total outage probability of x_1 is given by

$$P_{o,x_1} = P_{SD,x_1}\left(\gamma_{th}\right) P_{e2e,x_1}\left(\gamma_{th}\right). \tag{15}$$

Substituting (12) and (14) into (15) and after some algebraic manipulations, we can attain (9). Moreover, following similar steps in [13], the outage probability of x_2 in the S-D link, P_{o,x_2}, can be obtained as $P_{o,x_2} = 1 - P\left(\gamma_{D,x_1} > \gamma_{th}, \gamma_{D,x_2} > \gamma_{th}\right)$. With the help of (6) and (7), the proof is finished. ∎

To investigate the impact of fading parameters on the outage performance, we present the asymptotic outage probability expressions at the high SNR regime in Lemma 1 as follow.

Lemma 1. *The achievable diversity order of each symbol are the same as one.*

Proof: In the high-SNR regime, the outage probabilities of x_1 and x_2 can be approximated as

$$P_{o,x_1}^\infty = \frac{\gamma_{th}^2}{\left(\gamma_{th}\Omega_{SD} + \Omega_{RD}\right)\rho}, \tag{16}$$

$$P_{o,x_2}^\infty = \frac{\gamma_{th}\Omega_{SD}}{\gamma_{th}\Omega_{SD} + \Omega_{RD}} + \frac{\gamma_{th}\left(\Omega_{RD} + \Omega_{SD}\right)}{\rho\Omega_{SD}\left(\gamma_{th}\Omega_{SD} + \Omega_{RD}\right)}, \tag{17}$$

where we have used the approximation of $e^{-x} \approx 1 - x$ when $x \to 0$ [14, Eq. (1.211.1)]. The proof is concluded by defining the diversity order as $\lim_{\rho\to\infty} \frac{-\log P_o}{\log \rho} = 1$. ∎

It is remarkable that the high-SNR outage probability of x_1, P_{o,x_1}^∞, is independent of Ω_{SR}, while P_{o,x_1}^∞ is an decreasing function of Ω_{SD} and Ω_{RD}. Note that the outage probability of x_2 is lower bounded by a fixed value of $\gamma_{th}\Omega_{SD}/(\gamma_{th}\Omega_{SD} + \Omega_{RD})$ as $\rho \to \infty$. The outage performance can be improved by adopting a low threshold γ_{th}. Moreover, both outage probabilities are monotonically decreasing functions of the transmit SNR ρ. Unlike the legacy DF scheme where the diversity order is two, the diversity order of our proposed fixed DF based scheme is one. This is due to that fixed DF requiring the relay to fully decode the source information limits to the performance of direct transmission between the source and relay [4].

B. Achievable Rate

Achievable rate is a key performance metric for wireless communication systems, so we now focus on the achievable rate performance of the proposed relay assisted cooperative NOMA systems. According to the NOMA and fixed DF relay scheme, the total achievable rate of x_1 is given by

$$R_{x_1} = \underbrace{\frac{1}{2}\min\left\{\log_2\left(1 + \gamma_{D,x_1}\right), \log_2\left(1 + \gamma_{SR,x_1}\right)\right\}}_{R_{C,x_1}}$$
$$+ \underbrace{\frac{1}{2}\log_2\left(1 + \gamma_{SD,x_1}\right)}_{R_{D,x_1}}, \tag{18}$$

where R_{C,x_1} denotes the achievable rate of x_1 transmitted through R, and R_{D,x_1} denotes the achievable rate of x_1 transmitted directly from S to D. The parameter $1/2$ is due to using two time slots in relaying systems.

By substituting (1) into (18), we can readily derive R_{D,x_1} as

$$R_{D,x_1} = \frac{1}{2\ln 2 \rho\Omega_{SD}} \int_0^\infty \exp\left(-\frac{x}{\rho\Omega_{SD}}\right) \ln\left(1 + x\right) dx$$
$$= -\frac{1}{2\ln 2} \exp\left(\frac{1}{\rho\Omega_{SD}}\right) \text{Ei}\left(\frac{-1}{\rho\Omega_{SD}}\right), \tag{19}$$

where $\text{Ei}(\cdot)$ is the exponential integral function [14, Eq. (8.211.1)], and we have used the integral identity [14, Eq. (4.337.2)]. The achievable rate of x_2 can be derived as $R_{x_2} = R_{D,x_1}$ by using the same method to derive x_1. Moreover, we evaluate R_{C,x_1} by presenting the following Theorem 2.

Theorem 2. *For the relay assisted cooperative NOMA system, the approximated achievable rate of x_1 transmitted through R*

can be obtained as

$$R_{C,x_1} \approx \frac{1}{2\ln 2} \sum_{k=1}^{N} \frac{w_k \Omega_{RD} e^{-\frac{x_k}{\rho}\left(\frac{1}{\Omega_{RD}} + \frac{1}{\Omega_{SR}}\right)}}{(1+x_k)(x_k \Omega_{SD} + \Omega_{RD})}, \qquad (20)$$

where N is the number of integration points, the abscissas x_k and the weights w_k are defined as

$$x_k = \tan\left[\frac{\pi}{4}\cos\left(\frac{2k-1}{2N}\pi\right) + \frac{\pi}{4}\right], \qquad (21)$$

$$w_k = \frac{\pi^2 \sin\left(\frac{2k-1}{2N}\pi\right)}{4N\cos^2\left[\frac{\pi}{4}\cos\left(\frac{2k-1}{2N}\pi\right) + \frac{\pi}{4}\right]}. \qquad (22)$$

Proof: Please see Appendix. ∎

Although the expression of R_{c,x_1} presented in Theorem 2 is given in terms of sum series, we only need a few terms (e.g., $N < 60$) as verified by extensive Monte-Carlo simulations to get a satisfactory accuracy (e.g., smaller than 10^{-6}) for all considered cases. Combining the aforementioned results (19) and (20), we derive the total achievable rate of x_1 and x_2 as

$$R \approx \frac{1}{2\ln 2}\left(\sum_{k=1}^{N} \frac{w_k \Omega_{RD} e^{-\frac{x_k}{\rho}\left(\frac{1}{\Omega_{RD}} + \frac{1}{\Omega_{SR}}\right)}}{(1+x_k)(x_k \Omega_{SD} + \Omega_{RD})}\right.$$
$$\left. - e^{\frac{1}{\rho\Omega_{SD}}} \texttt{Ei}\left(\frac{-1}{\rho\Omega_{SD}}\right)\right). \qquad (23)$$

Because the exact analytical results above cannot directly provide physical insights, we now focus on the high-SNR regime and present the Lemma 2 as follows.

Lemma 2. *In the high-SNR regime, the asymptotic achievable rate of* (23) *is given by*

$$R^\infty = \log_2 \rho - \frac{E_c}{\ln 2} + \log_2 \Omega_{SD}$$
$$+ \frac{1}{2\ln 2}\sum_{k=1}^{N} \frac{w_k \Omega_{RD}}{(1+x_k)(x_k \Omega_{SD} + \Omega_{RD})}. \qquad (24)$$

Proof: As $\rho \to \infty$, we can apply the approximations of $\texttt{Ei}(-x) \approx E_c + \ln x$ [14, Eq. (8.212.1)] and $e^x \approx 1 + x$ when $x \to 0$ [14, Eq. (1.211.1)] on (23). After some algebraic manipulations, (24) can be obtained to finish the proof. ∎

From (24), it is obvious that, as the transmit SNR ρ increases, the achievable rate R^∞ improves. Moreover, a higher value of Ω_{SD}/Ω_{RD} will decrease the achievable rate, which means that the short distance of the R-D link benefits the rate performance of the relay assisted cooperative NOMA system. It is worthy to mention that the proposed scheme can achieve the rate scaling of $\log_2 \rho$, while the rate scaling of conventional CRS-OMA [4] and CRS-NOMA [11] is $\frac{\log_2 \rho}{2}$. This advantage is obtained by exploiting the second time slot and the NOMA principle in the proposed scheme. in the second time slot, the source S can send symbol x_2 to the destination D. Thanks to the NOMA principle, the destination D can decode symbols x_1 and x_2 by employing the SIC principle.

IV. NUMERICAL RESULTS

In this section, we provide numerical results to validate the outage probability and achievable rate results obtained
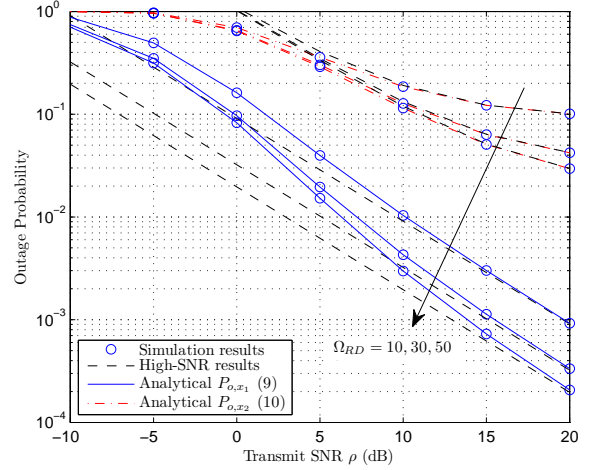

Fig. 2. Outage probability against the transmit SNR ρ, where different values of Ω_{RD} are considered but Ω_{SR} is fixed as 10 and γ_{th} is fixed as 0 dB).
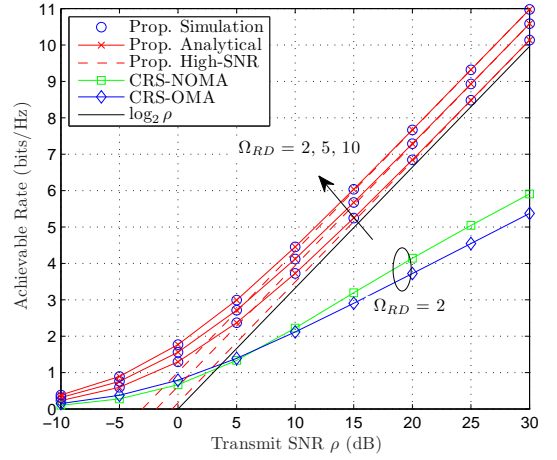

Fig. 3. Achievable rate against the transmit SNR ρ, where different values of Ω_{RD} are considered but Ω_{SR} is fixed as 10.

in Section III. We also provide some insights into the system performance. To validate the accuracy of the aforementioned expressions, comparisons with complementary Monte Carlo-simulated performance results are also presented. More specifically, 10^6 realizations of Rayleigh distribution random variables are generated. The distances of all links can be represented by the average power. Furthermore, a typical simulation scenario illustrated in Fig. 1 is considered with an normalized distance parameter of the $S-D$ link as $\Omega_{SD} = 1$.

The outage probability performance is presented in Fig. 2 against the transmit SNR ρ when different values of Ω_{RD} are considered. We can observe that the simulation results provide a perfect match to the analytical results obtained in Section III. In addition, as the quality of the R-D link increases, the outage probability P_{o,x_1} in (9) for the symbol x_1 also improves. However, the gap between the corresponding curves decreases as Ω_{RD} increases from 10, 30 to 50, which implies that its effect becomes less pronounced. Moreover, it is obvious from Fig. 2 that the high-SNR approximations are sufficiently

tight at moderate SNRs and become exact at high SNRs. Additionally, its accuracy improves for small values of Ω_{RD}. Note that P_{o,x_2} reduces to a constant value related to Ω_{RD} as ρ increases, which means the outage probability of x_2 is limited to the average power of the R-D link. The diversity orders of x_1 and x_2 are one, which has been accurately predicted in Lemma 1.

Fig. 3 depicts simulation results, analytical result (23), and high-SNR approximation (24) of the achievable rate against the transmit SNR ρ for the relay assisted cooperative NOMA system. It can be observed that the analytical curves are consistent with the simulation ones. In the high-SNR regime, the asymptotic expression derived in (24) is well matched with the exact one proving this asymptotic approximation to be accurate enough, and becomes tight as Ω_{RD} increases. The proposed scheme is also compared to existing CRS-NOMA [11] and CRS-OMA [4], respectively. From Fig. 3, we can find that the proposed scheme provides outstanding performance gains over CRS-NOMA and CRS-OMA. This is because we use SIC at the destination, and enable the transmission of x_2 during the second phase. Thanks to the relay node, the fading gain of x_1 is also enhanced by shorting the distance of the R-D link (e.g., larger values of Ω_{RD}). Moreover, it is worth mentioning that the rate scaling of $\log_2 \rho$ is achieved by the proposed scheme, while CRS-NOMA and CRS-OMA can only achieve the rate scaling of $\frac{\log_2 \rho}{2}$. In contrast to CRS-NOMA, the proposed scheme can also achieve a higher achievable rate in the low-SNR regime.

V. CONCLUSION

In this paper, we have proposed a relay assisted cooperative NOMA scheme. Analytical expressions of outage probability and achievable sum rate were derived. More specifically, the outage probability was given in closed-form, while an efficient numerical expression for the achievable sum rate was also obtained. Furthermore, the high-SNR asymptotic expressions for both performance metrics were presented to get better insights into the impacts of system parameters on the performance. Our analysis validated that larger average power of the R-D link has a beneficial effect on system performance. Finally, the proposed scheme outperforms conventional CRS-NOMA and CRS-OMA schemes in terms of achievable sum rate in whole SNR regime.

APPENDIX

The achievable rate R_{C,x_1} can be expressed as [15, Eq. (24)]

$$R_{C,x_1} = \frac{1}{2\ln 2} \int_0^\infty \ln(1+x) f_{C,x_1}(x)\, dx$$
$$= \frac{1}{2\ln 2} \int_0^\infty \frac{1-F_{C,x_1}(x_k)}{1+x}\, dx. \qquad (25)$$

Substituting (14) into (25), we have

$$R_{C,x_1} = \frac{1}{2\ln 2} \int_0^\infty \frac{1}{1+x} \frac{\Omega_{RD}}{x\Omega_{SD}+\Omega_{RD}}$$
$$\times \exp\left(-\frac{x}{\rho}\left(\frac{1}{\Omega_{RD}}+\frac{1}{\Omega_{SR}}\right)\right) dx. \qquad (26)$$

Unfortunately, the involving integral is very difficult to be solved in a closed form. However, by changing the variable of the integration in (25) as $x = \tan(\theta)$, R_{C,x_1} can be written as

$$R_{C,x_1} = \frac{1}{2\ln 2} \int_0^{\frac{\pi}{2}} \frac{1-F_{C,x_1}(\tan\theta)}{1+\tan\theta} \sec^2\theta\, d\theta. \qquad (27)$$

Then, we can use an efficient N-point Gauss-Chebyshev formula [16, Eq. (25.4.39)] to numerically derive

$$R_{C,x_1} \approx \frac{1}{2\ln 2} \sum_{k=1}^N w_k \frac{1-F_{C,x_1}(x_k)}{1+x_k}, \qquad (28)$$

where x_k and w_k are given by (21) and (22), respectively. Substituting (14) into (28), the proof can be concluded.

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