Parametric Velocity Synthetic Aperture Radar: Multilook Processing and Its Applications

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Abstract—Based on a parametric model and some optimum methods, it has been previously proved that parametric velocity synthetic aperture radar (VSAR) may improve the performances of moving target detection and parameter estimation simultaneously. In this paper, multilook processing is studied for parametric VSAR. At first, statistical signal models are established for azimuth multilook processing (AMLP) and range multilook processing (RMLP), respectively. By combining the multiple AMLP sublook pixel vectors, it is shown that the clutter parameter estimation accuracy can be further improved via the maximum-likelihood estimation methods. Meanwhile, based on the adaptive implementation for the optimum processing of VSAR, RMLP can be used to improve slowly moving target detection performance via the noncoherent integration of multiple range sublooks. Furthermore, it is shown that the Doppler frequencies of moving targets vary linearly with different range sublooks due to the different carrier frequencies of sublooks. Therefore, based on a proposed novel two-step multilook diversity (TS-MLD) estimator for RMLP, the “azimuth location ambiguity” of VSAR can be well resolved via least squares linear regression, without configuration change of the conventional VSAR. Also, the estimation accuracy of target’s unambiguous Doppler frequency, as well as target’s azimuth location, is derived for the proposed TS-MLD method. Finally, numerical experiments and scene simulations are provided to demonstrate the effectiveness of the proposed multilook-based methods.

Index Terms—Azimuth location ambiguity, maximum likelihood (ML), moving target detection, multilook processing, two-step multilook diversity (TS-MLD) method, velocity synthetic aperture radar (VSAR).

I. INTRODUCTION

It is a well-known difficult problem for synthetic aperture radar (SAR) to detect, locate, and image slowly moving targets in the strong ground clutter, which has attracted much attention [1]–[18]. In order to suppress the ground clutter and to better detect ground moving targets, some SAR systems based on multiple receiving channels have been proposed, e.g., space-time–frequency processing SAR [9], [10], multichannel SAR [11], velocity SAR (VSAR) [12], [13], and along-track interferometric SAR (AT-InSAR or ATI-SAR) [14]–[17]. Friedlander et al. introduced the concept of VSAR in [12] and [13]. Equipped with a linear array antenna, it is known that VSAR may simultaneously obtain multiple reflectivity images of the same scene. Meanwhile, via the fast Fourier transform (FFT) operators on the pixel vector sampled among multiple images, VSAR may not only detect the moving ground targets but also relocate them to true azimuth positions.

Nevertheless, the conventional VSAR has its own limitations due to the simple FFT operators. For example, VSAR normally uses a relatively sparse array [12], [13] to obtain the high frequency resolution and accuracy. Consequently, fast-moving targets may be easily ambiguous in the Doppler domain and may still be mislocated in the cross range after VSAR processing, which is called “azimuth location ambiguity” of VSAR. On the other hand, even though the large sparse array is used, the accuracies of parameter estimation, as well as the target location, are still unsatisfactory via FFT operators directly with the limited element samples. Lombardini et al. [17] introduced parametric methods for multibaseline AT-InSAR. In their studies, based on the parametric modeling of the sea-surface backscattering, super-resolution parametric methods were proposed for the joint estimation of model parameters, e.g., the averaged radial velocity and the coherence time of the sea-surface waves. It has been demonstrated that the estimation accuracy can be improved via the parametric methods.

In our previous paper [18], we have proposed a parametric signal model and optimal methods for VSAR which is called parametric VSAR. Parametric VSAR uses the adaptive implementation of the optimal processing (AIOP-VSAR) method and maximum-likelihood (ML) estimation methods for moving target detection and parameter estimation, respectively. It has been shown that the AIOP-VSAR can improve the detection performance in an inhomogeneous clutter background with the adaptive filtering. Furthermore, a moving target with blind speed in the zeroth channel can be detected via the AIOP-VSAR with certain improvement factor. In addition, based on the proposed ML methods, the estimation accuracies can approach the Cramer–Rao bounds (CRBs) for clutter parameters, as well as target location. Moreover, it has been proven that the proposed methods may effectively mitigate the “azimuth
location ambiguity” with a narrowly spaced array antenna and has the super-resolution ability to resolve “velocity layover” for multiple targets.

In this paper, a well-known SAR signal processing method, i.e., multilook processing, is further discussed for parametric VSAR. Normally, at the expense of spatial-resolution reduction, the multilook processing of SAR may suppress the image speckle and improve the radiation resolution to discriminate the scatterers with different backscattering intensities [2], [3]. Herein, we introduce the multilook processing to further improve the moving target detection and parameter estimation. First, parametric models are established for azimuth multilook processing (AML) and range multilook processing (RMLP), respectively. Compared to the full-look processing, it is known that the clutter-to-noise ratio (CNR) of azimuth sublook is reduced. However, it will be demonstrated that the clutter parameter estimation performance may be improved by combining all the multiple-sublook pixel vectors when the original CNR is not too low.

Furthermore, based on the AIOP-VSAR of parametric VSAR [18], it is found that the improvement factor in the clutter Doppler frequency area can be approximately constant in terms of range sublook number. Therefore, the detection performance of a slowly moving and small target can be improved via the noncoherent integration among multiple range sublooks. Moreover, due to different carrier frequencies of different range sublooks, it is shown that the Doppler frequencies of a moving target may vary slightly but linearly as a function of range sublook number. Therefore, based on the least squares (LS) estimator and the ML method, the multiple estimated Doppler frequencies may approach a line as a function of range sublook’s carrier frequencies, and the problem of “azimuth location ambiguity” can be well resolved. Unfortunately, the performance of direct linear regression result may be unsatisfactory, specifically for the high-frequency-band SAR. Thus, a novel “two-step multilook diversity (TS-MLD)” estimator is further provided, which uses the first step to estimate the ambiguous integer and use the second step to acquire the ultimate unambiguous Doppler frequency. It is shown that the TS-MLD method may resolve “azimuth location ambiguity” with high accuracy and the correct probability approaching to one. Furthermore, compared with the conventional VSAR, no hardware configuration change is required by TS-MLD. In addition, the high estimation accuracy of unambiguous target’s Doppler frequency, as well as its azimuth location, is also derived in this paper.

This paper is organized as follows. In Section II, the parametric multilook model is established for AMLP and RMLP, respectively. In Section III, the performance analysis of clutter parameter estimation is given based on AMLP. In Section IV, the noncoherent integration of multiple sublooks is discussed based on RMLP, and the TS-MLD method is proposed. In Section V, numerical experiments and scene simulations are presented. In Section VI, some conclusions are drawn.

II. PARAMETRIC VSAR MULTILOOK PROCESSING SIGNAL MODELS

In this section, based on the parametric VSAR studied in [18], we give the novel parametric models of thermal noise, distributed clutter, and isolated targets for AMLP and RMLP, respectively. As shown in Fig. 1, the X-axis is the cross-range (azimuth) direction, whereas the Y-axis is the range direction. The sampling times of the X- and Y-axes are denoted as τ and t, respectively. VSAR is equipped with a linear antenna array, including M receiving elements which are equally spaced along the flight track with interval d. The zeroth element also is a single transmitting element. The radar platform flies along the azimuth direction at altitude h with a constant forward velocity \( v_x \). Assume that the first element of the antenna array is located at \( x = 0 \) and a specified moving point \( P \) is located at \( (x_0, y_0) \) with slant range \( R_0 = \sqrt{x_0^2 + y_0^2 + h^2} \) when \( t = 0 \). Target \( P \) keeps constant azimuth-direction velocity \( v_x \) and range-direction velocity \( v_y \) in the total imaging duration. For the radar system, \( \lambda \) and \( f_c \) represent the wavelength and the carrier frequency, respectively. During the radar operation, the VSAR sequentially transmits the linear frequency modulation (LFM) pulses with time duration \( T_p \) and bandwidth \( B_r \), and then, the echoes are simultaneously received by the M elements that may reconstruct the M reflectivity images of the same scene.
A. Proposed Parametric VSAR Model

In the previous paper [18], we have established a parametric statistical model to decompose pixel vector among multi-images into three isolated components, namely, moving target, clutter of background, and thermal noise. The target vector $\mathbf{T}$ may be represented as

$$
\mathbf{T} = \sum_{i=1}^{N} \mathbf{S}_{Ti} = \sum_{i=1}^{N} \sigma_{Ti} e^{j\phi_{Ti}} \mathbf{a}(f_{di})
$$

where $f_{di} = -(2(x_0v_s + y_0v_y))/(\lambda R_0)$ denotes the Doppler frequency in the multiple-image domain of a specified target, $v_s$ represents the sampling time delay, $(\cdot)^T$ denotes the matrix transpose operation, and the target vector $\mathbf{T}$ is composed of $N$ target components $\mathbf{S}_{Ti}$, $i = 1, \ldots, N$, which are parameterized by scattering amplitude $\sigma_{Ti}$, phase $\phi_{Ti}$, and Doppler frequency $f_{di}$. Normally, $N > 1$ is satisfied in most area of the imaging scene. For a scenario with dense targets, $N > 1$, i.e., the “velocity layover” effect, may occur. In this paper, $N$ is assumed known for simplicity. Moreover, the relationship between the Doppler and azimuth shift $\Delta_s$ of the moving target can be given as

$$
\Delta_s = -\frac{R_0\lambda}{2v_s} f_{di}.
$$

Furthermore, the ML method [18] had been proposed by parametric VSAR to estimate the target Doppler frequency and relocate the moving target to its true azimuth position. Also, for the scattering of random rough background terrain in a certain pixel, the clutter model $\mathbf{C}$ has been established as an $M \times 1$ pixel vector

$$
\mathbf{C} = \mathbf{a}(f_{dc}) \odot \mathbf{c}
$$

where $f_{dc}$ is the clutter Doppler center regardless of platform motion, $\odot$ is the Hadamard product, and the vector $\mathbf{c}$ describes the “speckle” effect among the pixels of multi-images. Assuming that the number of randomly distributed scatterers in each pixel may be very large for the homogeneous background, $\sigma_c(i)$ and $\phi_c(i)$, where $i = 1, \ldots, M$, are modeled as the Rayleigh- and uniform-distributed random variables, respectively. Combining (1) and (3), the total pixel sampling vector $\mathbf{X}$ among elements may be totally expressed as

$$
\mathbf{X} = \mathbf{T} + \mathbf{C} + \mathbf{v}
$$

where the thermal noise vector $\mathbf{v}$ is modeled as a realization of a zero-mean white complex Gaussian vector process with covariance matrix $\sigma_v^2 \mathbf{I}$, where $\sigma_v^2$ denotes the noise variance and $\mathbf{I}$ denotes the identity matrix. For the pixel where only the clutter exists, (4) may be simplified as $\mathbf{X} = \mathbf{C} + \mathbf{v}$. Furthermore, the central limit theorem can be applied to model $\mathbf{C} + \mathbf{v}$ as a zero-mean complex Gaussian vector process with covariance matrix

$$
\mathbf{R} = \sigma_v^2 \mathbf{A}(f_{dc}) \mathbf{C} \mathbf{A}^H(f_{dc}) + \sigma_v^2 \mathbf{I}
$$

where $(\cdot)^H$ represents matrix conjugate (Hermitian) transpose, $\mathbf{A}(f_{dc})$ is an $M \times M$ diagonal matrix with diagonal elements of vector $\mathbf{a}(f_{dc})$, i.e., $\mathbf{A}(f_{dc}) = \text{diag}(\mathbf{a}(f_{dc}))$, and $\mathbf{C}$ is an $M \times M$ Hermitian matrix that is, in general, of full rank and only related to $\rho_f$ [31]. Under the clutter’s Gaussian autocorrelation assumption [8], [27], [28], we rewrite (5) as

$$
[R]_{i,j} = \sigma_v^2 \exp(j2\pi f_{dc} \Delta_{ij})
$$

and

$$
\delta_{ij} = \begin{cases} 
1, & i = j \\
0, & i \neq j 
\end{cases}
$$

where $f_{dc}$ and $\rho_f$ are the clutter’s Doppler center and spread, respectively, $\Delta_{ij} = (i-j)t_d$, and $\delta_{ij} = \begin{cases} 
1, & i = j \\
0, & i \neq j 
\end{cases}$. Under the aforementioned conditions, the problem is converted into the target detection and parameter estimation in a stationary Gaussian process. In addition, the pixel vector of VSAR is distributed as

$$
\mathbf{X} \sim \mathcal{CN}(\mathbf{m}_s, \mathbf{R})
$$

where $\mathcal{CN}(\cdot, \cdot)$ represents a multivariate complex Gaussian distribution with specified mean and covariance. Obviously, the target component in the proposed model is entirely deterministic and contributes to the mean part. Meanwhile, the clutter plus noise is a stationary random process and contributes to the covariance matrix part. Normally, the tasks of VSAR are as follows: first, to determine whether a moving target exists in a certain pixel vector, and second, based on models (4)–(8), we wish to accurately estimate the unknown parameters $\mathbf{X} = \left[f_{dc}, \rho_f, \sigma_c^2, \sigma_v^2, \sigma_c, f_{T_1}, f_{T_2}, \sigma_{T_N}, f_{T_N}, \phi_{T_N}\right]^T$ from the measurement $\mathbf{X}$. The parameters of interest are the clutter Doppler center $f_{dc}$, the clutter Doppler spread $\rho_f$, and the target Doppler frequency $f_{di}$, $i = 1, \ldots, N$, whereas the clutter amplitude $\sigma_c$, the noise amplitude $\sigma_v$, and the target amplitude $\sigma_{T_i}$, $i = 1, \ldots, N$, are the nuisance parameters.

B. Azimuth Multilook Parametric VSAR Model

Normally, the AMLP is implemented in combination with the SAR azimuth focusing, when the signals of both clutter and targets may be approximated as LFM signals as a function of slow time $t$. For the AMLP in the frequency domain, the Doppler spectrum is uniformly divided into $L_a$ sublooks along the Doppler frequency $f_1$ (the Fourier transform in terms of $t$), as shown in Fig. 2, where $B_a$ represents the clutter Doppler spread among $f_1$ in the signal domain, and it is mainly decided by the platform motion. With the estimated clutter
Doppler parameters, e.g., the clutter Doppler center $f_{t,dc}$ and
the Doppler rate, the matched reference signals are formed for
the $L_a$ sublooks. Notably, there are important differences
between $f_{dc}$ and $f_{t,dc}$. Symbol $f_{dc}$ describes the clutter Doppler
center in the pixel vector, which reflects the clutter internal
motion regardless of the platform motion. Meanwhile, $f_{t,dc}$
gives the description of the Doppler distribution in the SAR
signal domain, which is jointly decided by the radar beam direc-
tion, platform motion, and clutter internal motion. In Fig. 3(a),
the signal processing flowchart is given based on the typical
SAR range–Doppler imaging algorithm. Accordingly, the $M$
elements of linear array antenna may reconstruct $L_a$ subimages
as Fig. 3(b), respectively. For single azimuth sublook, only
$1/L_a$ pulse samplings of full look are coherently integrated for
clutter, and $1/L_a$ pulse samplings are noncoherently integrated
for noise. As for the moving target with small azimuth-direction
velocity $v_x$, its azimuth “defocusing” effect may be omitted
in the image domain, and there are $1/L_a$ pulse samplings of
full look that are partially coherently integrated in each pixel
sampling vector. Therefore, the $L_a$ pixel vectors of a certain
pixel may be approximated as (9), shown at the bottom of
the page, where $\sigma_{ei}, l = 1, \ldots, L_a$, represent $L_a$
realizations of $\sigma$, and they are independent and identically distributed
(i.i.d.) random vectors, whereas $v_l, l = 1, \ldots, L_a$, represent
$L_a$ realizations of $v$, and they are also i.i.d. random vectors.
Therefore, the statistical signal model of $L_a$ may be given as
(10)–(12), shown at the bottom of the page. From (12), it is
shown that the mean of the azimuth-sublook pixel vector is
uncertain due to the uncertain Doppler shift of target, whereas
the clutter component of the $L_a$ sublooks are i.i.d. samples.

C. Parametric RMLP Model of VSAR

It is known that the echoes of both clutter and targets may
be approximated as LFM signals as a function of fast time $\tau$.
The spectrum may be uniformly divided into $L_r$ sublooks along
frequency $f_r$ (the Fourier transform in terms of $\tau$), as shown
in Fig. 4, where $B_r$ represents the frequency bandwidth of
the transmitted signal. Therefore, the RMLP of VSAR may
be realized on the radar receiver subsystem, where the wideband
SAR echoes are filtered and demodulated into the $L_r$ channels
of subband signals, and each channel may generate a range

\[
X(l) \approx T(l) + C(l) + v(l), \quad l = 1, \ldots, L_a
\]
\[
T(l) \approx \left\{ \begin{array}{ll}
\sum_{i=1}^{N} \sigma T_{i} e^{j\phi a(f_{dc})}, & \text{when the target is in the current azimuth sublook} \\
0, & \text{when the target is not in the current azimuth sublook}
\end{array} \right.
\]
\[
C(l) = \frac{1}{L_a} a(f_{dc}) \bigotimes \sigma_{ei}
\]
\[
v(l) = \frac{v_l}{\sqrt{L_a}}
\]

\[
X(l) \sim CN(\mathbf{m}_{s,L_a}, \mathbf{R}_{L_a}), \quad l = 1, \ldots, L_a
\]
\[
|\mathbf{R}_{L_a}|_{i,j=1}^{K} = \frac{\sigma_r^2}{L_a} \exp(j2\pi f_{dc} \Delta_{ij}) \exp \left( \frac{-(2\pi \rho_f \Delta_{ij})^2}{2} \right) + \frac{\sigma_m^2}{L_a} \delta_{ij}
\]
\[
\mathbf{m}_{s,L_a} = \left\{ \begin{array}{ll}
\frac{1}{L_a} \sum_{i=1}^{N} \sigma T_{i} e^{j\phi a(f_{dc})}, & \text{when the target is contained in the } l \text{th sublooks} \\
0, & \text{when the target is not contained in the } l \text{th sublooks}
\end{array} \right.
\]
sublook image via the successive imaging method. Also, the RMLP may be realized by combining the range pulse compression, and the matched reference functions can be formed for the $L_r$ sublooks, respectively. In Fig. 5(a), the signal processing flowchart is given based on the typical range–Doppler imaging algorithm for $L_r$ channel subband signals. Accordingly, the $M$ elements of linear array antenna may, respectively, reconstruct $L_r$ subimages as in Fig. 5(b). For single range sublook, because...
only $1/L_r$ pulses of full look are coherently integrated for target and clutter and $1/L_r$ pulse energy is noncoherently integrated for noise, the $L_r$ pixel vectors may be given as

$$X(l) = T(l) + C(l) + v(l), \quad l = 1, \ldots, L_r$$

$$T(l) = \frac{1}{L_r} \sigma_T e^{j \phi_l} a(f_{dl})$$

$$C(l) = \frac{1}{L_r} a(f_{dc}) \sigma_c$$

$$v(l) = \sqrt{\frac{v}{L_r}}$$

where the sublook-dependent target Doppler frequency $f_{dl}$, $l = 1, \ldots, L_r$, may be given as

$$f_{dl} = \frac{2(x_0 v_x + y_0 v_y)}{\lambda_l R_0} = \frac{2(x_0 v_x + y_0 v_y) f_{cl}}{c R_0} = \frac{\lambda}{\lambda_l} f_d$$

and $f_{cl} = f_c + (((2l - 1 - L_r)B_r)/(2L_r))$ and $\lambda_l = c/f_{cl}$, $l = 1, \ldots, L_r$, represent the carrier frequency and wavelength for the $l$th sublook, respectively. Also, the clutter Doppler center $f_{dc}$ and spread $\rho_f$ may vary with range sublooks as

$$f_{dc} = \frac{\lambda}{\lambda_l} f_{dc}, \quad \rho_f = \frac{\lambda}{\lambda_l} \rho_{fi}. \quad (15)$$

Therefore, a statistical VSAR signal model of the RMLP may be given as

$$X(l) \sim CN(m_{n,L_r}(l), R_{L_r}(l)) \quad l = 1, \ldots, L_r \quad (16)$$

$$|R_{L_r}(l)|_{i,j=1}^K = \frac{\sigma_c^2}{L_r^2} \exp(j2\pi f_{dc} \Delta_{ij}) \times \exp\left(-\frac{(2\pi \rho_f \Delta_{ij})^2}{2}\right) + \frac{\sigma_v^2}{L_r} \delta_{ij} \quad (17)$$

$$m_{n,L_r}(l) = \frac{1}{L_r} \sum_{i=1}^N \sigma_T e^{j \phi_n} a(f_{di,i}) \quad (18)$$
D. Properties of the Multilook Parametric VSAR Model

From the proposed VSAR models for RMLP and AMLP, several properties can be given as follows.

1) The clutter components, i.e., \( C(l) \), \( l = 1, \ldots, L_a \), in the azimuth multilook VSAR processing are i.i.d. random vectors.

2) Due to the uncertain Doppler shift of target, the target components, i.e., \( T(l) \), \( l = 1, \ldots, L_a \), in the AMLP are also uncertain. For the subLooks with the target, \( T(l) \) can be approximately as i.i.d. random vectors. For the subLooks without the target, \( T(l) \) are approximately equal to zeros.

3) Due to the linearly varied Doppler center and spread as a function of sublooks as (15), the clutter components, i.e., \( C(l) \), \( l = 1, \ldots, L_r \), in the RMLP are not the i.i.d. random vectors.

4) Due to the linearly changed carrier frequency as a function of range sublooks, the target components, i.e., \( T(l) \), \( l = 1, \ldots, L_r \), have the linearly varied Doppler frequencies as a function of sublook number. That is, they are sublook dependent.

5) From (9) and (13), it is shown that the CNR and signal-to-noise ratio (SNR) of the sublooks are reciprocal of sublook number. Meanwhile, the signal-clutter ratio (SCR) is constant with the change of sublook number.

III. INHERENT LIMITATIONS OF VSAR PARAMETER ESTIMATION BASED ON AMLP

Because multiple i.i.d. pixel vector \( \mathbf{C} + \mathbf{v} \) in (9)–(12) may be obtained for the background components via AMLP, it is possible to improve the estimation accuracy of clutter parameters. To obtain the performance analysis for the ML method, we give the CRB analysis based on the single sublook and multilook sampling vectors, respectively, which provides the performance bounds.

A. Inherent Limitations Based on Single Sublook Sampling

It is well known that the mean square error of any unbiased estimator for the \( n \)th parameter \( \chi_n \) is lower bounded by the \( nn \)th element of the inverse of the Fisher information matrix (FIM) \( \mathbf{J} \) [8] as

\[
E \left( (\widehat{\chi}_n - \chi_n)^2 \right) \geq \left[ \mathbf{J}^{-1} \right]_{nn}.
\]

Because the single azimuth sublook vector \( \mathbf{X}(l) \) is complex, circular, and Gaussian with mean \( \mathbf{m}_i, L_a \) and covariance matrix \( \mathbf{R}_{L_a} \) as (9), the FIM elements can be expressed [8, p. 525] as

\[
J_{mn} = [\mathbf{J}]_{mn} = \text{tr} \left\{ \frac{\partial \mathbf{R}_{L_a}(\chi)}{\partial \chi_m} \right\} \frac{\partial \mathbf{R}_{L_a}(\chi)}{\partial \chi_n}
\]

\[+ 2 \Re \{ \frac{\partial \mathbf{m}_i}{\partial \chi_m} \} \mathbf{R}_{L_a}^{-1}(\chi) \frac{\partial \mathbf{m}_i}{\partial \chi_n} \]

where \([\cdot]_{mn}\) stands for the \( mn \)th element of a matrix, \( \text{tr}(\cdot) \) is the matrix trace, and \( \Re\{\cdot\} \) is the real part of a complex number. Evaluating the derivatives of variance matrix and mean of single sampling as (11) and (12), respectively, and plugging them into (20) yield the FIM given by (21), shown at the bottom of the page.

The nonzero entries of the FIM based on full look may be found in [18, Appendix A]. In the case of AMLP, only the reduced SNR and CNR of sublook should be considered. Note the following block structure of the FIM:

\[
\mathbf{J} = \text{diag}(\mathbf{J}_0, \mathbf{J}_1, \ldots, \mathbf{J}_M)
\]

where

\[
\mathbf{J}_0 = \begin{bmatrix}
\mathbf{J}_{f_{dc}, f_{dc}} & \mathbf{J}_{f_{dc}, \rho_f} & \mathbf{J}_{f_{dc}, \sigma^2_f} & \mathbf{J}_{f_{dc}, \sigma^2_{\phi_i}} & \mathbf{J}_{f_{dc}, \sigma^2} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\mathbf{J}_{\rho_f, f_{dc}} & \mathbf{J}_{\rho_f, \rho_f} & \mathbf{J}_{\rho_f, \sigma^2_f} & \mathbf{J}_{\rho_f, \sigma^2_{\phi_i}} & \mathbf{J}_{\rho_f, \sigma^2} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\mathbf{J}_{\sigma^2_f, f_{dc}} & \mathbf{J}_{\sigma^2_f, \rho_f} & \mathbf{J}_{\sigma^2_f, \sigma^2_f} & \mathbf{J}_{\sigma^2_f, \sigma^2_{\phi_i}} & \mathbf{J}_{\sigma^2_f, \sigma^2} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\mathbf{J}_{\sigma^2_{\phi_i}, f_{dc}} & \mathbf{J}_{\sigma^2_{\phi_i}, \rho_f} & \mathbf{J}_{\sigma^2_{\phi_i}, \sigma^2_f} & \mathbf{J}_{\sigma^2_{\phi_i}, \sigma^2_{\phi_i}} & \mathbf{J}_{\sigma^2_{\phi_i}, \sigma^2} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{\sigma_{T_1}, \rho_f} & \mathbf{J}_{\sigma_{T_1}, \sigma^2_f} & \mathbf{J}_{\sigma_{T_1}, \sigma^2_{\phi_i}} & \mathbf{J}_{\sigma_{T_1}, \sigma^2} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{f_{dc}, \sigma^2_{\phi_i}} & \mathbf{J}_{f_{dc}, \sigma^2} & \mathbf{J}_{f_{dc}, \sigma^2_{\phi_i}} & \mathbf{J}_{f_{dc}, \sigma^2} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{\phi_i, f_{dc}} & \mathbf{J}_{\phi_i, \sigma^2_f} & \mathbf{J}_{\phi_i, \sigma^2_{\phi_i}} & \mathbf{J}_{\phi_i, \sigma^2} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{\sigma_{T_N}, \sigma_{T_N}} & \mathbf{J}_{\sigma_{T_N}, \sigma^2_f} & \mathbf{J}_{\sigma_{T_N}, \sigma^2_{\phi_i}} & \mathbf{J}_{\sigma_{T_N}, \sigma^2} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{f_{dc}, \sigma^2_{\phi_N}} & \mathbf{J}_{f_{dc}, \sigma^2} & \mathbf{J}_{f_{dc}, \sigma^2_{\phi_N}} & \mathbf{J}_{f_{dc}, \sigma^2} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{\phi_i, f_{dc}} & \mathbf{J}_{\phi_i, \sigma^2_f} & \mathbf{J}_{\phi_i, \sigma^2_{\phi_N}} & \mathbf{J}_{\phi_i, \sigma^2} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{\phi_N, f_{dc}} & \mathbf{J}_{\phi_N, \sigma^2_f} & \mathbf{J}_{\phi_N, \sigma^2_{\phi_N}} & \mathbf{J}_{\phi_N, \sigma^2} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{J}_{\phi_N, f_{dc}} & \mathbf{J}_{\phi_N, \sigma^2_f} & \mathbf{J}_{\phi_N, \sigma^2_{\phi_N}} & \mathbf{J}_{\phi_N, \sigma^2} & \cdots & 0 & 0
\end{bmatrix}
\]
reversed [8] as
\[ J^{-1} = \text{diag}(L_0, L_1, \ldots, L_M) \]  
(25)
\[ L_0 = \begin{bmatrix} J_{f_{dc}, f_{dc}} & J_{f_{dc}, \rho_f} & J_{f_{dc}, \sigma_d^2} & J_{f_{dc}, \sigma_v^2} \\ J_{\rho_f, f_{dc}} & J_{\rho_f, \rho_f} & J_{\rho_f, \sigma_d^2} & J_{\rho_f, \sigma_v^2} \\ J_{\sigma_d^2, f_{dc}} & J_{\sigma_d^2, \rho_f} & J_{\sigma_d^2, \sigma_d^2} & J_{\sigma_d^2, \sigma_v^2} \\ J_{\sigma_v^2, f_{dc}} & J_{\sigma_v^2, \rho_f} & J_{\sigma_v^2, \sigma_d^2} & J_{\sigma_v^2, \sigma_v^2} \end{bmatrix}^{-1} \]  
(26)
\[ L_i = \begin{bmatrix} J_{f_{dc}, f_{dc}} & J_{f_{dc}, \rho_f} & J_{f_{dc}, \sigma_d^2} & J_{f_{dc}, \sigma_v^2} \\ J_{\rho_f, f_{dc}} & J_{\rho_f, \rho_f} & J_{\rho_f, \sigma_d^2} & J_{\rho_f, \sigma_v^2} \\ J_{\sigma_d^2, f_{dc}} & J_{\sigma_d^2, \rho_f} & J_{\sigma_d^2, \sigma_d^2} & J_{\sigma_d^2, \sigma_v^2} \\ J_{\sigma_v^2, f_{dc}} & J_{\sigma_v^2, \rho_f} & J_{\sigma_v^2, \sigma_d^2} & J_{\sigma_v^2, \sigma_v^2} \end{bmatrix}^{-1}, \quad i = 1, \ldots, N. \]  
(27)

Consequently, the CRB for clutter parameters based on single sublook sampling can be represented as
\[ \text{CRB}_{\text{single}-\text{look}}(f_{dc}) = [L_0]_{11} \]  
(28)
\[ \text{CRB}_{\text{single}-\text{look}}(\rho_f) = [L_0]_{22}. \]  
(29)

B. Inherent Limitations Based on Multiple Azimuth Sublook Samplings

It is shown that \( L_0 \) i.i.d. samplings can be obtained via AMLP. Furthermore, the FIM of any sampling is diagonal as (21), and the elements of the “background” part in the overall FIM can thus be rewritten as
\[ [J_0]_{mn} = \sum_{l=1}^{L_a} [J_0(l)]_{mn}, \quad l = 1, \ldots, L_a. \]  
(30)
From (9)–(12), it is straightforward that
\[ J_0(l) = J_0, \quad l = 1, \ldots, L_a \]  
(31)
\[ \tilde{J}_0 = L_a J_0. \]  
(32)
Furthermore, the CRBs of \( f_{dc} \) and \( \rho_f \) based on multilook samplings can be derived as
\[ \text{CRB}_{\text{multilook}}(f_{dc}) = \text{CRB}_{\text{single}-\text{look}}(f_{dc})/L_a \]  
(33)
\[ \text{CRB}_{\text{multilook}}(\rho_f) = \text{CRB}_{\text{single}-\text{look}}(\rho_f)/L_a. \]  
(34)

C. Discussion of Clutter Parameter Estimation Based on AMLP

By now, it is still uncertain to answer whether the clutter parameter estimation accuracies may be improved based on AMLP. The uncertainty is because of the following two opposite factors that should be considered.

1) Based on the model (9), it is obvious that the CNR of azimuth sublook is linearly reduced as a function of sublook number \( L_a \). Furthermore, it is known [18] that the root-mean-square error bound (RMSEB), i.e., \( \text{RMSEB} = \text{CRB}^{1/2} \), of \( f_{dc} \) and \( \rho_f \) are all the monotonically decreasing functions of the CNR. Thus, the clutter parameter estimation accuracy of single sublook sampling can be reduced with the increase of \( L_a \).

2) Compared with single sublook, it is shown that the estimation accuracy of clutter parameters may be improved by combining the \( L_a \) i.i.d. \( C + v \) sublook samplings as (33) and (34).

Fortunately, the RMSEB as function of CNR (as the Fig. 8 in [18]) is a non-linear function of CNR. When the CNR is not too low, e.g., higher than 25 dB as the numerical experiment in [18], it is shown that the RMSEB may be changed slightly with the decrease of CNR. Therefore, by combining the multiple sublook vectors, it is possible to improve the estimation accuracies of clutter parameters as (33) and (34). That is, there exists an optimum \( L_a \), corresponding to a “local minimum” of the RMSEB, for clutter parameter estimation as long as the original CNR of the pixel vector is not too low. When the sublook number is less than the number corresponding to the “local minimum,” the performance is improved with the increase of \( L_a \). However, when the sublook number is larger than the “local minimum” of the RMSEB, the performance is inversely worse with the increase of \( L_a \).

IV. PARAMETRIC RMLP OF VSAR AND ITS APPLICATIONS

Different from the AMLP model (9), the target may be contained among all the range sublooks as (13). Thus, the target detection and parameter estimation are discussed for RMLP in this section.

A. Improvement for a Slowly Moving Target Based on Noncoherent Integration

Based on the proposed AIOP-VSAR method [18], it is known that the optimum weight for target detection is \( w = \mu R^{-1} S \), where \( \mu \) is a constant and \( S = a(f_d) \) is the Doppler vector of the target. Because \( a(f_d) \) is unknown before the target is correctly detected and accurately estimated, the Doppler filter banks in Fig. 6 with a group of searching Doppler frequencies are used, i.e.,
\[ w_i = \mu R^{-1} S_i, \quad i = 1, \ldots, M \]  
(35)
where
\[ S_i = \begin{bmatrix} 0, \exp\left(-j2\pi \frac{i}{M}\right), \exp\left(-j2\pi \frac{2i}{M}\right), \ldots, \\
\exp\left(-j2\pi \frac{(M-1)i}{M}\right) \end{bmatrix}^T, \quad i = 1, \ldots, M \]  
(36)
represent the \( i \)th searching Doppler frequency of the Doppler filter bank. Obviously, the structure in Fig. 6 may be quickly realized via the FFT operators, and the \( M \) outputs of the Doppler filter bank may be given as
\[ Y = \text{FFT}(R^{-1}X) \]  
(37)
where \( Y = [y_1, y_2, \ldots, y_M]^T \) denotes the output vector. It is known that the Doppler frequency resolution of Fourier transform is
\[ \rho_f = \frac{1}{M T_d} = \frac{2v_s}{M d}. \]  
(38)
Fig. 6. Doppler filter bank of the AIOP-VSAR.

Meanwhile, for the identical moving target, the Doppler frequency difference between the \( L_r \)th look and the first look is given as

\[
\Delta f_d = f_{d_{\text{max}}} - f_{d_{\text{min}}} = \frac{2(x_0 v_x + y_0 v_y)B_r}{eR_0}. \tag{39}
\]

Normally, we have the \( \Delta f_d \ll \rho f_d \) in accordance with the typical VSAR system parameters. For example, for a fast moving target located at \((x_0, y_0) = (0, 15\,000\, \text{m})\) with velocity \((v_x, v_y) = (1, 10\, \text{m/s})\), we have \( \rho f_d = 187.5\, \text{Hz} \) and \( \Delta f_d = 13.3\, \text{Hz} \). That is, based on RMLP, a moving target in the different sublooks may appear in the same channel of the Doppler filter bank. This fact implies that the target detection performance can be improved via the noncoherent integration among multiple sublooks, which is a well-used method [6], [27] for radar to improve the detection performance. Because only the amplitude information is extracted, the SNR gain may be less than the coherent integration. Normally, for the large \( L_r \), the improvement factor of the noncoherent integration may be approximated [27] as

\[
\text{IF}_{\text{NCI}} = \text{IF}_{\text{CI}} - L_{\text{NCI}}
\]

\[
= 10 \log_{10}(L_r) - L_{\text{NCI}} \approx 5 \log_{10}(L_r). \tag{40}
\]

where \( \text{IF}_{\text{CI}} = 10 \log 10(L_r) \) denotes the improvement factor of the coherent integration, which is proportional to the number of sublooks, and \( L_{\text{NCI}} \) denotes the noncoherent integration loss compared with the coherent integration.

On the other hand, it is obvious from (13) that SNR and CNR are all \( 1/L_r \) decreasing functions of range sublook number \( L_r \). In the nonclutter Doppler frequency area, the thermal noise is the unique factor to affect the target detection, and the signal-to-interference ratio (SIR) is equal to SNR. Therefore, based on the AIOP-VSAR, the SIR loss of a single sublook may be given as

\[
L_{\text{sublook}} = 10 \log_{10}(L_r). \tag{41}
\]

Thus, compared with the full-look processing, the improvement factor in the nonclutter Doppler frequency area is

\[
\text{IF}_{\text{NC}} = \text{IF}_{\text{NCI}} - L_{\text{sublook}} \approx -5 \log_{10}(L_r). \tag{42}
\]

Obviously, the target detection performance may be deteriorated in the nonclutter Doppler frequency area, even using the noncoherent integration among multiple range sublooks.

However, for the clutter Doppler channel in the clutter Doppler frequency area, the clutter may be much larger than noise, and we have \( \text{SIR} \approx \text{SCR} \). On the other hand, the SCR of each sublook may be assumed as a constant as full look as (13) with the simultaneously reduced signal and clutter. That is, we have \( L_{\text{sublook}} \approx 0 \) for the slowly moving target in the clutter Doppler frequency area. Therefore, it is possible to adopt the non-coherent integration by using the multiple approximately equal SIR sublooks to improve the output SIR in the clutter area. Then, the ultimate improvement factor in this channel \( \text{IF}_{\text{C}} \) may be given as

\[
\text{IF}_{\text{C}} = \text{IF}_{\text{NCI}} - L_{\text{sublook}} \approx 5 \log_{10}(L_r). \tag{43}
\]

By combining the noncoherent integration results as (42) and (43), we can give the answers to the question whether the noncoherent integration of RMLP may improve the target detection based on the AIOP-VSAR as follows.

1) For the fast-moving target in the nonclutter Doppler frequency area, the noncoherent integration among range multilooks cannot improve the target detection due to the linearly reduced SNR of sublook.

2) For the slowly moving target in the clutter Doppler frequency area, because the clutter is the main factor to affect the target detection and SCR is approximately constant as a function of \( L_r \), the target detection performance may be improved via the noncoherent integration of RMLP.

3) For the target with Doppler frequency between the clutter and the nonclutter Doppler frequency area, the target detection performance may be uncertain with the noncoherent integration gain and reduced SNR of sublook.

B. Resolving Azimuth Location Ambiguity Based on Range Multilook Diversity

To address “azimuth location ambiguity” of VSAR, we have proposed several conceptual VSAR systems based on the Chinese remainder theorem, e.g., multifrequency VSAR [19], dual-speed VSAR [20], nonuniform linear array SAR [21], bistatic linear array SAR [22], and minimum-redundancy linear array SAR [23], each of which may detect and locate both slowly and fast-moving targets simultaneously. Unfortunately, all the aforesaid systems may change the conventional VSAR configuration and increase the hardware complexity to some extent. Based on the ML estimation method [18] of parametric
VSAR, we also have shown that the azimuth location ambiguity may be dramatically mitigated by using a narrowly spaced array antenna. However, the antenna element space cannot be too small in real applications. In this paper, a novel method will be proposed based on RMLP, without configuration changes or increase of hardware complexity.

Based on the proposed ML method of parametric VSAR [18], a specific moving target’s ambiguous Doppler frequency as a function of \( L_r \) range sublooks, i.e., \( \widehat{f}_d, \), may be given as

\[
\begin{align*}
\widehat{f}_{d_1} &= f_{d_1} - N_A f_s + n_1 \\
\widehat{f}_{d_2} &= f_{d_2} - N_A f_s + n_2 \\
&\vdots \\
\widehat{f}_{d_{L_r}} &= f_{d_{L_r}} - N_A f_s + n_{L_r}
\end{align*}
\]  

(44)

where \( f_{d_l}, l = 1, \ldots, L_r, \) represent the target’s real Doppler frequency as a function of \( L_r \) range sublooks as (14), and \( N_A \) is an unknown ambiguous integer. \( n_l, l = 1, \ldots, L_r, \) represent the estimation errors of \( L_r \) range sublooks, respectively, \( f_s = 1/t_d = 2v_s/d \) is the Doppler sampling frequency of the pixel vector.

Based on the ML estimators of parametric VSAR [18], the estimation error of each range sublook \( n_l, l = 1, \ldots, L_r, \) may be assumed as the Gaussian-distributed variable as \( n_l \sim N(0, \sigma_{CRB,f_d}^2), l = 1, \ldots, L_r. \) Because the RMSEB varies slowly in terms of the target Doppler frequency (e.g., [18, Fig. 10]), we have

\[
\sigma_{CRB,f_d}^2 \approx \cdots \approx \sigma_{CRB,f_{d_{L_r}}}^2 = \sigma_{CRB,f_d}^2
\]

with \( \widehat{f}_{d_1} \approx \widehat{f}_{d_2} \approx \cdots \approx \widehat{f}_{d_{L_r}}. \) Therefore, \( n_l, l = 1, \ldots, L_r, \) may be approximated as

\[
n_l \sim N(0, \sigma_{CRB,f_d}^2), \quad l = 1, \ldots, L_r
\]  

(45)

where

\[
\mathcal{T}_{dc} = \text{mod} \left( \frac{2(x_0v_x + y_0v_y)f_c}{cR_0}, f_s \right)
\]  

(46)

represents the averaged ambiguous Doppler frequency of carrier frequency \( f_c. \) Then, (44) may also be rewritten as

\[
\begin{align*}
\widehat{f}_{d_1} &= k_d f_{c_1} - N_A f_s + n_1 \\
\widehat{f}_{d_2} &= k_d f_{c_2} - N_A f_s + n_2 \\
&\vdots \\
\widehat{f}_{d_{L_r}} &= k_d f_{c_{L_r}} - N_A f_s + n_{L_r}
\end{align*}
\]  

(47)

where \( c \) is the light speed, \( f_{c_l}, l = 1, \ldots, L_r, \) denote the central frequencies of the \( L_r \) range sublooks, and \( k_d = (2(x_0v_x + y_0v_y)/cR_0) \) represents a constant Doppler rate in terms of carrier frequency. Obviously, the estimation of unknown parameters \( k_d \) and \( N_A \) is a linear regression problem, and the minimum LS estimator of \( k_d \) may be given as

\[
k_{d,LS} = \frac{\sum_{l=1}^{L_r} (f_{c_l} - f_c)(\widehat{f}_{d_l} - \mathcal{T}_d)}{\sum_{l=1}^{L_r} (f_{c_l} - f_c)^2}
\]  

(48)

where

\[
\mathcal{T}_d = \frac{1}{L_r} \sum_{l=1}^{L_r} \widehat{f}_{d_l}
\]  

(49)

denotes the averaged ambiguous Doppler frequency of \( L_r \) range sublooks. It can be proven that

\[
k_{d,LS} \sim N \left( k_d, \frac{\sigma_{CRB,f_d}^2}{\sum_{l=1}^{L_r} (f_{c_l} - f_c)^2} \right)
\]  

(50)

Obviously, the expectation and the variance of Gaussian-distributed \( k_{d,LS} \) are \( k_d \) and \( \sigma_{CRB,f_d}^2/\sum_{l=1}^{L_r} (f_{c_l} - f_c)^2, \) respectively. Then, with the estimated \( k_{d,LS}, \) the unambiguous Doppler frequency \( f_d \) can be directly obtained as

\[
\widehat{f}_d = k_{d,LS} f_c.
\]  

(51)

Because the radar carrier frequency \( f_c \) is a known constant and \( k_{d,LS} \) is Gaussian distributed as (50), it is straightforward that the random estimator \( \widehat{f}_d \) may also be Gaussian distributed as

\[
\widehat{f}_d \sim N \left( k_f c, \xi \sigma_{CRB,f_d}^2 \mathcal{T}_{dc} \right)
\]  

(52)

where \( \xi = f_c^2/\sum_{l=1}^{L_r} (f_{c_l} - f_c)^2. \) Obviously, (51) may provide the unambiguous target Doppler frequency, whose expectation is the correct target Doppler frequency \( f_d \) with the variance \( \xi \sigma_{CRB,f_d}^2. \) Unfortunately, \( \xi \) may be too large for VSAR with the high carrier frequency and limited multilook number. For example, for Ku-band SAR with \( f_c = 15.4 \) GHz and \( L_r = 8, \) we have \( \xi = 8024, \) which implies that the RMSEB of \( \widehat{f}_d \) may be about 93 times that of ambiguous Doppler frequencies \( \widehat{f}_{d_l}, \) \( l = 1, \ldots, L_r, \) of the ML methods. Also, the ultimate location accuracy of VSAR may be deteriorated for about 93 times, which is unacceptable for real use.

Let us define \( a = N_A f_s, \) where \( a \) can be estimated as

\[
a = \mathcal{T}_d - k_{d,LS} f_c.
\]  

(53)

With (46)–(52), it is straightforward that \( a \) is also Gaussian distributed as

\[
a \sim N \left( N_A f_s, \left( \frac{1}{L_r} + \xi \right) \sigma_{CRB,f_d}^2 \mathcal{T}_{dc} \right).
\]  

(54)

Meanwhile, the estimation of ambiguous Doppler frequency can be given as \( \widehat{f}_{d_{fc}} = \widehat{f}_d. \) From the definition of \( \widehat{f}_d \) as (49), we have

\[
\widehat{f}_{d_{fc}} \sim N \left( k_f c - N_A f_s, \frac{1}{L_r} \sigma_{CRB,f_d}^2 \mathcal{T}_{dc} \right).
\]  

(55)

From (55), it is shown that the RMSEB of \( \widehat{f}_{d_{fc}} \) may be \((1/\sqrt{L_r})\) times that of ambiguous Doppler frequencies \( \widehat{f}_{d_l}, \) \( l = 1, \ldots, L_r. \) That is, the accuracy of \( \widehat{f}_{d_{fc}} \) may be remarkably improved with RMLP. Therefore, if we can combine the ambiguous frequency \( \widehat{f}_{d_{fc}} \) and the correct ambiguous integer \( N_A, \)
an excellent estimate of the target Doppler frequency, called “TS-MLD,” can be formed as

$$\tilde{f}_d = \text{round}(\tilde{\theta}/f_s) f_s + \tilde{d}_{dc}$$  \hspace{1cm} (56)$$

where round(\(\)) is the round function to obtain the ambiguous integer. In (56), a kind of two-step strategy is adopted to obtain the target Doppler frequency, which uses the first step to obtain the ambiguous integer and the second step to acquire the ultimate unambiguous Doppler frequency. Obviously, as long as round(\(\tilde{\theta}/f_s\)) \(= N_A\), the estimator (56) has the expectation \(\tilde{f}_d\) and RMSEB \(1/\sqrt{L_r}\sigma_{\text{CRB},f_{dc}}\). That is, compared with the result (51), the estimate accuracy can be improved \(\sqrt{L_r}\xi\) times from (56). By now, the last questions are what the probability of round(\(\tilde{\theta}/f_s\)) \(= N_A\) is and how to improve this probability. With (55), this probability may be given as

$$P = \frac{N_A f_s + f_s/2}{N_A f_s - f_s/2} \exp\left(-\frac{(t - N_A f_s)^2}{2(L_r + \xi)}\frac{1}{\sigma_{\text{CRB},f_{dc}}^2}\right) dt$$

$$= \Phi\left(\frac{f_s/2}{\sqrt{1/L_r + \xi^2} \sigma_{\text{CRB},f_{dc}}}\right) - \Phi\left(\frac{-f_s/2}{\sqrt{1/L_r + \xi^2} \sigma_{\text{CRB},f_{dc}}^2}\right)$$  \hspace{1cm} (57)$$

where \(\Phi(u) = \int_{-\infty}^{u} \exp\left(-\frac{u^2}{2}\right) du\). It is known that when \(u > 3.1\), the probability of \(\Phi(u)\), i.e., the cumulated probability of standard Gaussian distribution, may approach 0.998. That is, when

$$f_s > 6.2\sqrt{1/L_r + \xi^2} \sigma_{\text{CRB},f_{dc}}$$  \hspace{1cm} (58)$$

Equation (56) may approach \(f_d\) with RMSEB \(1/\sqrt{L_r}\sigma_{\text{CRB},f_{dc}}\) at the probability larger than 99.8%. Fortunately, because \(f_s = 2v_s/d\) is large for the narrowly spaced array antenna and \(\sigma_{\text{CRB},f_{dc}}\) is also low via the proposed ML method, (58) can be easily satisfied for the proposed parametric VSAR. Also, according to (2), we have

$$\tilde{\Delta}_s = -\frac{R_0 \lambda}{2v_s} \tilde{f}_d$$  \hspace{1cm} (59)$$

where \(\tilde{f}_d\) and \(\tilde{\Delta}_s\) are the estimated Doppler frequency and the azimuth shift of the moving target, respectively. In addition, (59) is also Gaussian distributed as

$$\tilde{\Delta} \sim N\left(\Delta_s, \frac{1}{L_r} \sigma_{\text{CRB},\Delta}^2\right)$$  \hspace{1cm} (60)$$

where

$$\sigma_{\text{CRB},\Delta}^2 = \frac{R_0^2 \lambda^2}{4v_s^2} \sigma_{\text{CRB},f_{dc}}^2$$  \hspace{1cm} (61)$$

From (55) and (59), it is shown that the RMSEB of target’s Doppler frequency and location can be further improved for nearly \(\sqrt{L_r}\) times, compared with the result of the full-look parametric VSAR [18].

Fig. 7. Clutter parameter estimation accuracy as a function of azimuth sublook number. (a) High CNR case (CNR = 35 dB). (b) Low CNR case (CNR = 5 dB).

V. NUMERICAL EXPERIMENTS AND SCENE SIMULATIONS

In this section, we will verify the proposed methods by numerical experiments and scene simulations. First, we give a conceptual airborne Ku-band parametric VSAR with system parameters and a nominal scenario as in [18, Section 4, Table 1], which is omitted in this paper for simplicity.

A. Improvement for the Clutter Parameter Estimation via AMLP

To demonstrate the clutter parameter estimation performance as a function of the azimuth range multilook number, we design two numerical experiments. In the first experiment, no system parameter is changed. In the second experiment, the original CNR of full-look is changed from 35 to 5 dB, which may occur in the case of far-range SAR. Then, the high and low CNR curves of clutter estimation as a function of sublook number are shown in Fig. 7(a) and (b), respectively. Obviously, in the high CNR case, there is a relatively large area as the dashed box shows, where the clutter estimation accuracies are improved with the increase of the azimuth sublook number. That is,
although the CNR of each sublook pixel vector is reduced, the ultimate estimation performance can be improved because the number of azimuth pixel vectors is increased. Because the RMSEB function has a “local minimal” when \( L_a = 8 \) as Fig. 7(a), it is shown that \( L_a = 8 \) may be an optimum azimuth sub-look number for the clutter parameter estimation. On the contrary, for the low CNR case as in Fig. 7(b), the clutter estimation accuracies are deteriorated with the increase of the range sublook number. That is, although the number of i.i.d. pixel vectors is increased, the ultimate estimation performance is deteriorated because the CNR of each vector is too low.

**B. Improvement for Slowly Moving Target Detection via Noncoherent Integration**

In this section, it will be demonstrated that the RMLP may improve the slowly moving target detection. Herein, a wooded area with 20-kn wind velocity is considered, with the clutter velocity spread \( \rho_v = 0.22 \) m/s (\( \rho_f = 22.53 \) Hz) being from [35, Ch. 15, pp. 9]. Based on the AIOP-VSAR, Fig. 8(a) and (b) shows the improvement factor curves of RMLP with and without noncoherent integration, respectively. From Fig. 8(a), we can see that the improvement factor can be reduced in both the clutter and the nonclutter Doppler frequency area. However, via the noncoherent integration among the multiple range sublooks, the improvement factor can be increased and exceed the full-look case \( (L_r = 1) \), as shown in Fig. 8(b), in the clutter Doppler frequency area. That is, the detection performance of the slowly moving target can be improved with the proposed noncoherent integration based on RMLP.

**C. Resolving Azimuth Location Ambiguity Based on Range Multilook Diversity**

In this section, we will demonstrate an improved performance of TS-MLD. In Fig. 9, the correct de-ambiguity probability is plotted as a function of the Doppler sampling frequency \( f_s \). It is shown that when \( f_s > 200 \) Hz, the probability may approach one. That is, the element space \( d \) should be less than 1.5 m to meet the sampling requirement, which is satisfied by our proposed parametric VSAR with \( d = 0.1 \) m [18]. In Fig. 10(a) and (b), there is a moving target located at \((x_0, y_0) = (0, 15000) \) m with a velocity vector \((v_x, v_y) = (0, 18) \) m/s, and the Monte Carlo trials of RMLP with \( L_r = 8 \) are repeated for 100 times. Because the \( f_s = 1500 \) Hz and Doppler frequency \( f_d = 1812.1 \) Hz, its ambiguous number is equal to one, and
the ambiguous Doppler frequency is equal to 312.1 Hz. The results of TS-MLD and that of (51) are simultaneously shown in Fig. 10(a); it is obvious that both the two estimators may obtain the unambiguous Doppler frequency with expectation $f_d = 1812.1$ Hz. Nevertheless, the TS-MLD as (56) significantly outperforms the direct LS estimator as (51) from Fig. 10(b). Based on the statistical evaluation of the Monte Carlo experiments, the RMSEB of (51) and TS-MLD are 39.7305 and 0.1525 Hz, respectively, which are perfectly consistent with (52) and (55). Therefore, compared with that of (51), more than 261 times higher accuracy may be obtained by TS-MLD. In other words, the TS-MLD may obtain the unambiguous Doppler frequency estimation with high accuracy and high reliability.

D. Scene Simulation

In this section, we provide a scene simulation to verify the TS-MLD method. The primary system parameters are the same as those in [18, Table 1]. The range pixel space $l_r = 0.5$ m, and the azimuth pixel space is also interpolated to $l_x = 0.5$ m. The SAR platform flights along the $X$-axis from left to right with a uniform velocity. Furthermore, the scene area is assumed as $512 \times 512$ m, and the scene center is located at $(x, y) = (0, 15000)$ m, which is composed of 16 subareas with
different backscattering intensities. In the center of this scene, there is a direct road represented as a black strip from top to down, as shown in Fig. 11, on which some moving targets are moving with uniform velocities. In the simulation, the detected “targets” are relocated to the “true” position based on their estimated Doppler frequency. For the conventional VSAR, the element space $d$ may be increased to improve the Doppler frequency accuracy and resolution, and $d = 1.6$ m is used. Therefore, the Doppler frequency bin $\Delta f_d = 2v_x/(dM)$ is $11.72$ Hz, and the unambiguous Doppler frequency area is $[-93.75, 93.75]$ Hz. Suppose that there is a moving target located at $(x_0, y_0) = (0, 15.128$ m) with a velocity vector $(v_x, v_y) = (0, -1.99$ m/s), the Doppler frequency of the moving target is $f_d = 200.4$ Hz, and its azimuth shift is $\Delta = -200.7$ m. When the target is added into the scene, the image is shown in Fig. 11(a). Meanwhile, also with an element space $d = 1.6$ m, we use the proposed TS-MLD for comparison; the target’s unambiguous Doppler frequency can be correctly obtained. Therefore, the moving target can be correctly relocated as Fig. 11(b) via the proposed TS-MLD method. However, for the conventional VSAR, the target’s velocity is still ambiguous and mislocated as Fig. 11(b).

VI. CONCLUSION

In this paper, multilook processing has been investigated for parametric VSAR to further improve the target detection and parameter estimation. At first, the statistical signal models are established for AMLP and RMLP, respectively. Then, it is demonstrated that the clutter parameter estimation can be improved via the AMLP as long as the original CNR is not too small. Also, it can improve the detection performance for slowly moving targets via the RMLP in the clutter Doppler frequency area, based on the noncoherent integration among multiple sublooks. Furthermore, the TS-MLD method is proposed based on the RMLP, which may well resolve the azimuth location ambiguity without configuration change of the conventional VSAR. Finally, numerical experiments and scene simulations are provided to demonstrate the effectiveness of the proposed methods.

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