Fractional sparse energy representation method for ISAR imaging

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Abstract: Inverse synthetic aperture radar (ISAR) is an effective radar imaging technology to obtain high-resolution images of manoeuvring targets. To improve the cross-range resolution of ISAR images, a fractional sparse energy representation method is proposed for ISAR imaging of rotating targets by combining fractional Fourier transform with sparse representation technique. Experimental results demonstrate that the proposed algorithms can efficiently realise high-quality imaging in addition to robustness against noise.

1 Introduction

Inverse synthetic aperture radar (ISAR) imaging is well known for its advantages of achieving high-resolution images of manoeuvring targets [1–5]. Conventional range-Doppler (RD) ISAR imaging algorithms [6, 7] are based on the assumption that during the coherent processing interval (CPI), the Doppler frequency corresponding to each scatterer remains constant and the high cross-range resolution is dependent on the long CPI. However, the rotational motion of the target in a long CPI may cause serious migration through resolution cells and varying reflectivity characteristics with respect to the change of a viewing angle. This results in considerable difficulty to obtain well-focused ISAR images.

A short CPI can certainly simplify the ISAR imaging model because the rotation rate of the target can be considered as a constant in this case [8, 9]. However, it is challenging to obtain high cross-range resolution from the short CPI data. In [10, 11], modern spectral estimation methods are utilised to obtain high-resolution ISAR images. However, the ISAR signal is supposed to be approximately stationary in these methods. Some sparsity representation algorithms have also been proposed in [12, 13] to improve the cross-range resolution. However, the basic assumption of the algorithms in [12, 13] is that the Doppler frequency corresponding to each scatterer remains constant during the CPI. Ignoring the time-varying Doppler factor simplifies the process of ISAR imaging but may result in degeneration of the image quality.

Time-frequency analysis algorithms are used to produce range-instantaneous Doppler ISAR images. Usually, range compression is performed over the ISAR echo signal. Then after slant range alignment and motion compensation, we can apply time-frequency transform to each range bin. Thus, the instantaneous ISAR image of the target in the azimuth direction can be obtained by taking the slices in the slow-time domain. In [14, 15], the short-time Fourier transform (STFT) is applied to obtain the instantaneous ISAR image. However, the STFT cannot achieve high time-resolution and high frequency-resolution simultaneously. The most popular high-resolution time-frequency analysis method is the Wigner–Ville distribution (WVD) [15, 16]. However, the WVD suffers from the well-known cross-term interference problem. Some low-pass filters are combined with WVD to suppress the cross-term, such as the smoothed pseudo-WVD algorithm [17] and modified WVD algorithm [18]. However, there usually exists a compromise between the cross-term suppression performance and the time-frequency resolution. Fractional Fourier transform (FrFT) is an effective time-frequency analysis tool for chirp signals. FrFT has the excellent energy concentrating ability to chirp signals [19, 20], and it has attracted great interest in recent years [21–23]. This particular energy concentrating property appears sparse in the fractional domain. The inherent sparsity motivates us to combine FrFT with sparse representation.

In this study, we synthesise the merits of FrFT and sparse representation technique and propose a new ISAR imaging method called fractional sparse energy representation method. The range profile is assumed to be obtained via the conventional pulse compression method and we aim to improve the cross-range resolution for short CPI data with time-varying Doppler frequencies. After slant range migration correction, the cross-range signal in each range bin is considered as a summation of some chirp signals with an unknown chirp rate. Then the fractional energy representation of the cross-range signal can be obtained with a few time-fractional-frequency atoms by exploiting the sparse property. Hence, the instantaneous ISAR image of the target in the azimuth direction can be retrieved by taking the slices in the cross-fractional frequency domain, and the chirp rate parameters can be retrieved via the matched-fractional order. On the one hand, benefiting from the structural sparsity of the chirp signal in the fractional domain, the proposed method can achieve high-resolution ISAR imaging performance compared with conventional time-frequency analysis based on FT. On the other hand, compared with the traditional sparse representation [24, 25], the fractional sparse energy representation method is more flexible for chirp signals thanks to the additional transform angle of FrFT. Finally, the experimental results with the b727 data [26] demonstrate that the superiority of the proposed algorithm over the other state-of-the-art ISAR imaging methods based on time-frequency analysis.

The rest of the paper is organised as follows. In Section 2, we introduce the ISAR signal model. The proposed method of retrieving both the ISAR image and the chirp rate parameters is formulated in Section 3. Experimental results based on both simulated and real data are presented in Section 4. Finally, the concluding remarks are given in Section 5.

2 ISAR signal model

The target motions can be classified as two kinds of components: translational motion and rotational motion. In this study, it is assumed that the translational motion has been compensated as done in [27, 28]. The rotation rate of the target can be considered as a constant during the short CPI, i.e. the target is uniformly...
rotating. In this case, the rotation model of a dominant scatterer $P_l$ located at position $(x_0, y_0)$ is illustrated in Fig. 1, where $\omega_0$ is the rotation rate and $P(t)$ is the position of a dominant scatterer at the slow time $t$. The instantaneous range migration of the scatterer $y(t)$ can be given as \[ y(t) = d \sin(\theta_0 + \Delta \theta) = d \sin(\theta_0 + \omega_0 t) \]

\[ = d \sin \theta_0 \cos \omega_0 t + d \cos \theta_0 \sin \omega_0 t \]

\[ = y_0 \cos \omega_0 t + x_0 \sin \omega_0 t. \] \hspace{1cm} (1)

Considering that the rotation angle $\Delta \theta = \omega_0 t$ is not very large during the short CPI, the following approximations are applicable:

\[ \sin \omega_0 t \simeq \omega_0 t, \]

\[ \cos \omega_0 t \simeq 1 - \frac{1}{2} (\omega_0 t)^2. \] \hspace{1cm} (2, 3)

Then (1) can be simplified as

\[ y(t) = y_0 \left[ 1 - \frac{1}{2} (\omega_0 t)^2 \right] + x_0 \omega_0 t. \] \hspace{1cm} (4)

After the slant range migration correlation, the Doppler phase of the signal reflected from the scatterer can be expressed as

\[ \varphi(t) = -\frac{4\pi}{\lambda} R(t) = -\frac{4\pi}{\lambda} [R_0 + y(t)] \]

\[ = -\frac{4\pi}{\lambda} [R_0 + y_0 \left[ 1 - \frac{1}{2} (\omega_0 t)^2 \right] + x_0 \omega_0 t] \]

\[ = -\frac{4\pi}{\lambda} [R_0 + y_0 + x_0 \omega_0 t - \frac{y_0}{2} (\omega_0 t)^2]. \] \hspace{1cm} (5)

where $\lambda$ is the wavelength, $R(t)$ and $R_0$ are the ranges from the radar to the scatterer and to the rotation centre, respectively. From (5), it is clear that the signal in the $l$th range bin is a chirp signal and the chirp rate $\Gamma_l$ is

\[ \Gamma_l = \frac{2y_0 \omega_0^2}{\lambda} = \frac{2(b + l\Delta r)\omega_0^2}{\lambda} = \Gamma_0 + \beta, \] \hspace{1cm} (6)

where $l$ is the range bin index, $\Delta r$ denotes the range gap between two successive range bins, $\Gamma_0$ denotes the chirp rate of the initial range bin, and

\[ \beta = 2\Delta r \omega_0^2 / \lambda. \] \hspace{1cm} (7)

From (6), it is clear that (i) there is a linear relationship between the chirp rate and the slant range index with a slope $\beta$, and (ii) the chirp rate $\Gamma_l$ is a common parameter for all scatterers in the $l$th range bin. In general, a real target, such as an aircraft or a ship, contains more than one dominant scatterer. Thus, after the range alignment, the model of the cross-range signal in the $l$th range bin can be characterised as a summation of several chirp signals

\[ s(t) = \sum_{i=1}^{m} \sigma_i \exp \left[ 2\pi f_{\delta_i} t + \frac{1}{2} \Gamma_i^2 t^2 \right], \quad t \leq T_l. \] \hspace{1cm} (8)

where $\mu_i$ denotes the number of dominant scatterers in the $l$th range bin, $\sigma_i$ denotes the proportional to the $l$th dominant scatterer reflectivity, $f_{\delta_i}$ denotes the Doppler frequency of the $l$th dominant scatterer, and $T$ is the observation time.

3 Proposed algorithm

In this section, we propose a novel fractional sparse energy representation algorithm for ISAR imaging. Firstly, we formulate the algorithm for ISAR imaging of non-cooperative rotating targets. Then, some specific problems are discussed.

3.1 Algorithm description

The FrFT of signal $s(t)$ is defined as [19]

\[ S_\alpha(u) = F^\alpha \{ s(t) \}(u) = \int_{-\infty}^{+\infty} s(t) K_\alpha(t, u) dt, \] \hspace{1cm} (9)

where $\alpha$ is the transform angle, $F^\alpha$ is the FrFT operator, $p = 2\alpha/\pi$ is the order of FrFT, and the kernel function $K_\alpha(t, u)$ is indicated by

\[ K_\alpha(t, u) = \sqrt{\frac{1 - j\cot \alpha}{2\pi}} \exp \left[ j \pi (u^2 + \alpha) (\cot \alpha - \csc \alpha) t \right], \quad \alpha \neq k \pi, \]

\[ \delta(t - 0), \quad \alpha = 2k \pi, \]

\[ \delta(t + 0), \quad \alpha = (2k + 1) \pi. \] \hspace{1cm} (10)

where $k$ is an integer and $\delta(\cdot)$ is the Dirac Delta function.

According to the signal model (8), FrFT of the $l$th range bin ISAR echo signal can be represented

![ISAR geometry](image-url)
where \( A_0 = \sqrt{\arccot(1 - j \cot \alpha) / 2 \pi} \). When \( \alpha = \arccot(-2 \pi f) \Delta = \alpha_0 \) i.e. in the best fractional frequency domain, (11) can be rewritten as

\[
S_0(u) = 2 A_0 T e^{j(\pi/2) \cot \alpha} \sum_{i=0}^{\mu-1} \sigma_i \sin(e^{j(2\pi f_0 - \alpha \sigma_i)}),
\]

(12)

From (12), the FrFT is obtained as a summation of several sinc functions with peaks in the fractional frequency domain and the number of sinc functions is theoretically \( \mu \). It means that the FrFT has the capability of concentrating a wideband chirp signal into a few coefficients.

In what follows, we apply the adaptive time-frequency transform [29] defined by a matching pursuit with a dictionary of Gabor functions to (12). Firstly, we construct a complete Gabor dictionary as follows:

\[
\mathcal{D} = \left\{ g_i(t) \left| g_i(t) = \frac{1}{\sqrt{\sigma_i}} e^{j(\pi/2) \sigma_i t} \right; \sigma = (\sigma, \eta, \xi) \in \mathbb{R}^+ \times \mathbb{R}^2 \right\},
\]

(13)

where \( g(t) = e^{j(\pi/2) \alpha \sigma^2} \) is the Gaussian function, \( \sigma > 0 \) and \( \xi \) denote scale, translation, and fractional frequency modulating, respectively. The factor \( 1/\sqrt{\sigma} \) normalises the norm of \( g_i(t) \) to 1.

According to the matching pursuit method [29], for the FrFT \( S_0(u) \) of signal model (8), an appropriate countable subset of orthogonal unit atoms \( \{ g_i(t) \}_i \in \mathcal{D} \) can be selected such that

\[
S_0(u) = \sum_{i=0}^{\mu_0} \left( R S_{0r} S_i \right) g_i(u),
\]

(14)

and

\[
\| S_0 \|^2 = \sum_{i=0}^{\mu_0} \left( R S_{0r} S_i \right)^* \left( R S_{0r} S_i \right),
\]

(15)

where the residue \( R S_{0r} \) is defined as

\[
R S_{0r}(u) = \begin{cases} R^{-1} S_0(u) - \sum_{i=0}^{\mu_0} \left( R S_{0r} S_i \right) g_i(u), & i \geq 1, \\ S_0(u), & i = 0. \end{cases}
\]

(16)

Thus the adaptive energy density of \( S_0(u) \) can be expressed as

\[
E S_0(u, v) = \sum_{i=0}^{\mu_0} \left( R S_{0r} S_i \right)^* W g_i(u, v),
\]

(17)

where \( W g_i(u, v) \) is the WVD of \( g_i(u) \), \( u \) and \( v \) denote fractional frequency and cross-fractional frequency, respectively.

Furthermore, according to (12), the FrFT with the best transform angle of the ISAR signal is a summation of several sinc functions with peaks. Hence there exists a subset of orthogonal unit atoms \( \{ g_i(t) \}_{i=0, 1, ..., \mu - 1} \subset \mathcal{D} \) such that

\[
\left( R S_{0r} S_i \right) g_i(u) \approx 2 A_0 T e^{j(\pi/2) \cot \alpha} \left( \frac{2\pi f_0}{\csc \alpha} - u \right),
\]

(18)

This means that we could use several atoms to approximate (12), and we have

\[
\sum_{i=0}^{\mu-1} \left( R S_{0r} S_i \right)^* g_i(u) > \| R S_{0r} \|^2.
\]

(19)

Then the following approximations hold:

\[
\| S_0 \|^2 \geq \sum_{i=0}^{\mu_0} \left( R S_{0r} S_i \right)^* g_i(u),
\]

(20)

and

\[
S_0(u) \approx \sum_{i=0}^{\mu_0} \left( R S_{0r} S_i \right) g_i(u)
\]

(21)

Hence (17) can be rewritten as

\[
E S_0(u, v) = \sum_{i=0}^{\mu_0} \left( R S_{0r} S_i \right)^* W g_i(u, v)
\]

(22)

Thus we obtain the fractional sparse energy representation \( E S_0 \) of the ISAR signal in (8). From (22) it can be seen that the fractional sparse energy representation exhibits the energy concentrating property of the chirp signal. It preserves the peaks with much fewer side lobes than FrFT. Therefore, we can estimate the Doppler frequency more accurately by

\[
\hat{\gamma}_i = \arg \max_u \| S_0 \|_2,
\]

(23)

\[
\left\{ \hat{\theta}_i, i = 1, ..., \mu \right\} = \arg \max_u \| E S_0 \|_2,
\]

(24)

The flowchart of the proposed algorithm is shown in Fig. 2. According to the above description, the detailed steps of the proposed algorithm are outlined as follows:

Firstly, we construct a complete Gabor dictionary (13) by discretising \( \gamma = (\sigma, \eta, \xi) \). The approximate optimal discrete method is as follows [29]:

\[
\gamma_i = \left( 2^i, 2^{i-1} u, 2^i v \right), \quad i \in \mathbb{Z},
\]

(25)

where \( 0 < i \leq \log_2 N, 0 \leq a \leq N/2^{i+1}, 0 < b < 2^{i+1} \) and \( N \) usually equals the length of the signal to be processed. Thus the complete dictionary \( \mathcal{D} = \{ g_i \}_{i=1, \ldots, \log_2 N} \) is obtained.

According to signal model (8), after the slant range alignment, the cross-range signal in the \( l \)th range bin can be expressed as

\[
s_l(n) = \sum_{i=1}^m \sigma_i \exp \left[ 2\pi f_0 \Delta t + \frac{\pi f_0}{2} \Gamma_{l} \gamma_i \right],
\]

(26)

where \( n = 1, 2, \ldots, N, \)
where $\Delta t$ denotes the time interval and $N$ is the total number of pulses. Then according to (23), we obtain the best transform angle $\hat{\alpha}_l$ after performing FrFT to (26).

Then we select $B_l = \{g_{0,l}, l = 0, 1, \ldots, \mu_l - 1\} \subset \mathcal{D}$ via orthogonal matching pursuit (OMP) [30, 31] to satisfy

$$S_{\hat{g}_l}(m) = \sum_{i=0}^{\mu_l-1} \hat{a}_l g_{0,l}(m), \quad m = 1, \ldots, M,$$

where $S_{\hat{g}_l}(\cdot)$ is the FrFT with the best transform angle of (26), $M$ is the number of sampling points in the fractional frequency domain. Hence according to (22), the fractional sparse energy representation of (26) can be calculated as

$$ES_{\hat{g}_l}(m,k) = \sum_{i=0}^{\mu_l-1} \hat{a}_l W g_{0,l}(m,k),$$

$$m = 1, \ldots, M; \quad k = 1, \ldots, K,$$

where $m$ and $k$ denote the grid indices in the fractional frequency domain and cross-fractional frequency domain, respectively.

From (24), we can extract the Doppler frequency information for the $l$th range bin by taking slices in the cross-fractional frequency domain, i.e.

$$\theta_l(m) = ES_{\hat{g}_l}(m,k), \quad l = 1, \ldots, L,$$

where $k \in \{1, 2, \ldots, K\}$. Therefore the final ISAR image can be obtained by all $\theta_l$, i.e. $\Theta = \{\theta_1, \theta_2, \ldots, \theta_L\}$.

In addition, according to (24), we can estimate the chirp rate $\hat{\Gamma}_j$ for all $l = 1, \ldots, L$. Then according to (6), we can estimate the slope $\hat{\beta}$ between the chirp rate and the slant range index via Outliers’ deletion method [9]. From (7) the estimation of the rotation rate of the target can be directly obtained by

$$\hat{\Delta} = \sqrt{\hat{\beta}/(2\Delta t)}.$$

In summary, the procedures of the proposed algorithm are as follows:

1. initiate $l = 1$, construct an complete Gabor dictionary $\mathcal{D} = \{g_{0,l}\}$ according to (25), and calculate the WVD for every basis $g_{0,l}$;

2. calculate the FrFT $S_{\hat{g}_l} = [S_{\hat{g}_l}(1), \ldots, S_{\hat{g}_l}(M)]^T$ and chirp rate $\hat{\Gamma}_j$ according to (24);

3. choose $B_l = \{g_{0,l}, l = 0, 1, \ldots, \mu_l - 1\} \subset \mathcal{D}$ via OMP satisfying (27), meanwhile obtain the coefficients $\{b_0, \ldots, b_{\mu_l-1}\}$;

4. calculate the fractional sparse energy representation $ES_{\hat{g}_l}$ of (26) according to (28), and then extract the Doppler frequency information $\theta_l$ via (29);

5. set $l = l + 1$, repeat 2 and 3, until $l = L$;

6. the correct ISAR image is $\Theta = \{\theta_1, \theta_2, \ldots, \theta_L\}$, and the rotational rate of the target is estimated by (30).

3.2 Discussion

(i) The performance of the range resolution compared with FrFT: Usually, the bigger the bandwidth of the transmitted signal is, the better the range resolution is [32]. Both the FrFT and the proposed method are applied in cross-range data processing. Therefore, the range resolution of the proposed method is theoretically same as the FrFT. However, according to (12), the FrFT is obtained as a summation of several sinc functions in the fractional domain. There exist several sidelobes in each range bin. These sidelobes on an adjacent range bin may interfere with each other. Hence, along with the range axis, there is a little ambiguous in ISAR images. This means that the range resolution of the FrFT is slightly worse than the proposed algorithm. In addition, this is verified by experiments in Section 4.

(ii) The robustness against noise compared with FrFT: According to (8), the ISAR echo signal can be rewritten with additive noise $w(t)$ as

$$s(t) = \sum_{i=1}^{\mu} a_i e^{j2\pi f_{\theta_i} T} + w(t), \quad t \leq [T].$$

Then similar to (12), the FrFT of (31) with the best fractional angle $\alpha_l = \arccos(-2\Gamma_j T)$ can be expressed as

$$S_{\hat{w}}(u) = 2A_w \cos^2 T \sum_{i=1}^{\mu} a_i \text{sinc}[(2\pi f_{\theta_i} - \text{wsec} \alpha_l)T] + S_{\hat{w}}(u),$$

where $S_{\hat{w}}(u)$ is the FrFT of noise $w(t)$ with the fractional angle $\alpha_l$.

For the proposed algorithm, according to (18), in a certain range of signal-to-noise ratio (SNR) the fractional energy representation of (31) can be expressed as
\[ E_{\text{ISAR}}(u, v) = 4A_k^2T^2 \sum_{i=0}^{N-1} \left( \frac{2\pi f_{b,i}}{\csc \theta_{li}} - u \right) W_{\text{ISAR}}(u, v) \].

Compared with (32) and (33), in a certain range of SNRs, the effect of noise on FrFT is greater than on the proposed algorithm. This means that compared with FrFT, the proposed algorithm can improve the anti-noise performance.

4 Experimental results

In this section, experiments with synthetic and real data, generated by the Naval Research Laboratory, are used to demonstrate the performance of the proposed algorithm.

4.1 Experiments on synthetic data

The parameters of the simulated radar system are given in Table 1. The number of range cells in the range domain is \( L = 64 \) and 1256 pulses are collected. In this experiment, the range gap between two successive range bins is set to be equal to the range resolution. After the slant range migration correction, the cross-range signal at a range bin contains several chirp signals with the same chirp rate. Discretise the fractional frequency domain and cross-fractalional frequency domain into \( M = K = N \) grids, where \( N \) is the total number of cross-range cells at each range bin. Fig. 3 shows the time-frequency distribution of the cross-range signal in the 32nd range bin obtained by STFT, WVD, fractional WVD (FrWVD) [33] and the proposed method, respectively. The Doppler frequencies of the two main components of the signal decrease with the same slope, which is equal to the chirp rate. From Fig. 3a, it can be seen that the resolution of STFT is limited. We can get higher resolution by WVD as shown in Fig. 3b. However, there exists cross-term interference. To accumulate the energy of the chirp signal better, the results of performing FrWVD on the echo signal is shown in Fig. 3c. Similar to WVD, FrWVD also suffers from the cross-term interference problem. From Fig. 3d, we can see that the proposed method has eliminated the cross-term interference while maintaining high time-frequency resolution.

The proposed algorithm is applied to retrieve the ISAR image and the chirp rate of the ISAR signal in each range bin. In what follows, the comparisons with the state-of-the-art ISAR imaging algorithms based on time-frequency analysis, such as STFT in [14], WVD in [16] and FrFT in [21], are conducted to demonstrate the superior performance of the proposed method. Fig. 4 shows the ISAR imaging results obtained by using the conventional RD algorithm. The result obtained by using the STFT is shown in Fig. 5a. It can be seen that the image is blurred due to the poor resolution and high sidelobe. Fig. 5b presents the imaging result obtained by WVD. Compared with STFT, the resolution of the image obtained by WVD is improved, i.e. the image resolution in Fig. 5b is much higher than that in Fig. 5a. However, the cross-term interference problem of WVD is too serious to retrieve the image correctly. Thanks to the good concentration ability of chirp signals in the fractional frequency domain, imaging results based on FrFT are well focused, as shown in Fig. 5c. One can see that the performance of cross-term interference reduction of FrFT is better than STFT and WVD. However, according to (12), the FrFT of the echo signal is obtained as a summation of several sinc functions in the fractional domain. Therefore, there exists several sidelobes in each range bin. These sidelobes on the adjacent range bin may interfere with each other. Hence, in Fig. 5c, the range resolution cell is a little ambiguous. Fig. 5d shows the ISAR images reconstructed by the proposed method. The range sidelobe of the proposed algorithm is improved compared with FrFT. Thus, Section 3.2(i) is verified. In particular, Fig. 6 is obtained by the proposed method only using 60% of measurements. Compared with the conventional RD, STFT, WVD, and FrFT methods, the proposed method is much better in terms of ISAR image focusing with limited measurements.

In addition, according to (6) and (23), the estimation of the slope is about \( \hat{\beta} = -4.015 \) m/s. Consequently, the rotation rate estimation is \( \hat{\omega}_b = 14.821 \) rad/s according to (30).

![Table 1 System model parameters for simulated data](image)

In what follows, the performances of the different algorithms versus SNRs are tested. The target scene and the imaging results obtained by the method based on the STFT for different SNRs are shown in Figs. 7a and 8a. It can be seen that the imaging results dramatically degrade with the decrease of the input SNR. In Figs. 7b and 8b, the imaging results obtained under different SNRs by the method based on WVD are shown. Compared with STFT, WVD has certain noise tolerance ability since the coherent characteristic of the adjacent pulse is enhanced. However, it suffers from the cross-term interference problem. From Figs. 7c and 8c, the imaging results obtained from the method based on FrFT are presented. It can be seen that the images are well focused when SNR = −5 dB. When SNR = −10 dB, the imaging result is too noisy. In contrast, the proposed method can still produce satisfactory results as shown in Figs. 7d and 8d. Although some unwanted artificial points occur around the dominant scatterers due to the increased noise energy level, the resulting ISAR images produced by the proposed algorithm are still acceptable for target recognition and classification.
The image contrast [34] is used to measure the quality of focusing. The expression is

\[ C = \frac{\sqrt{\langle |I_{nm}|^2 \rangle - \langle |I_{nm}| \rangle^2}}{\langle |I_{nm}| \rangle} \]  

(34)

where \( I_{nm} \) is the complex ISAR image, \( \langle \cdot \rangle \) calculates the spatial mean. A high focusing quality is related to a large value of contrast. In addition to the visual results, the contrast obtained by RD, a method based on STFT in [14], WVD in [16], FrFT in [21] and the proposed algorithm against different SNRs are given in Table 2. The indicator contrasts as a function of SNR for the ISAR image obtained by RD, a method based on STFT, WVD, FrFT and the proposed algorithm is plotted in Fig. 9. It is observed that the contrast of imaging results obtained by a method based on WVD is higher than STFT. However, in general, the contrasts of imaging results obtained by STFT and WVD are both at a low level. The contrasts of images obtained by a method based on FrFT are much higher than STFT and WVD for different SNRs, as shown in Fig. 9. However, the image contrasts of FrFT decrease significantly with the decrease of the input SNR. On the contrary, the contrasts of images obtained by the proposed algorithm maintain a high level and change slightly with the varying SNR. This means that the proposed method is robust in the presence of noise.
Fig. 7  Comparison of ISAR images under the situation of SNR = −5 dB
(a) STFT method in [14], (b) WVD method in [16], (c) FrFT method in [21], (d) Proposed method

Fig. 8  Comparison of ISAR images under the situation of SNR = −10 dB
(a) STFT method in [14], (b) WVD method in [16], (c) FrFT method in [21], (d) Proposed method
4.2 Experiment on real data

In this section, the real b727 data is applied to further validate the proposed method. The radar system parameters are given in Table 3.

After the range compression, as illustrated in Figs. 10 and 11, we use the conventional RD, a method based on STFT, WVD, FrFT and the proposed methods to process the cross-range profile, respectively. We discretise the fractional frequency domain and the cross-fractional frequency domain into $M = K = N$ grids, where $N$ is the total number of cross-range grids of each range bin. In addition, the chirp rates of ISAR signals at all range bins are estimated by the proposed method.

The imaging performances of different methods using real data are comparable as shown in Figs. 10 and 11. One can see that the proposed algorithm retrieves the aircraft shape well and has fewer sidelobes. The estimation slope $\hat{\beta}$ is about $-0.4814 \text{s}^{-2}$. According to (30), the rotation rate of the target is $\hat{\omega}_0 = 5.1323 \text{°}/\text{s}$.

According to [34], the image contrast with the reference image for quantitatively evaluating the quality of focusing on the ISAR images is provided in Table 4. As indicated in Table 4, the images

<p>| Table 2 Performance evaluation by contrast under different SNRs |</p>
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Fig. 9 Contrast as a function of the SNR for the ISAR image obtained by RD, a method based on STFT in [14], WVD in [16], FrFT in [21] and the proposed method

| Table 3 System model parameters for simulated data |
|----------------------------------|---------------|---------------|---------------|---------------|---------------|
| Type of radar | Stepped frequency |
| carrier frequency $f_c$ | 9 GHz |
| bandwidth $B$ | 150 MHz |
| range resolution $R_r = C/2B$ | 1 m |
| PRF | 20 kHz |

Fig. 10 ISAR imaging results of b727 airplane obtained by conventional RD
obtained by using the proposed method maintain a higher contrast than those given by other methods mentioned above.

5 Conclusion

FrFT and sparse representation are two effective techniques in the field of ISAR imaging. This study proposes a new fractional sparse energy representation method for ISAR imaging of uniformly rotating targets by introducing sparse representation technique into the fractional time-frequency analysis. The received ISAR echo is decomposed as a summation of several chirp signals. After the slant range migration alignment, the fractional energy representation of the cross-range signal can be obtained with a few time-fractional-frequency atoms due to the sparse structure. Hence, the instantaneous ISAR image of the target in the azimuth direction can be retrieved by taking the slices in the cross-fractional frequency domain. The proposed algorithm achieves significant performance on high-resolution ISAR imaging by exploiting the strong spatial sparsity of chirp signals in the time-fractional-frequency domain. The experiments on both simulations and real data demonstrate the superior performance of the proposed algorithm over existing time-frequency based ISAR imaging methods.

6 Acknowledgment

This work was supported in part by the National Natural Science Foundation of China (grant nos. 61331021, 61421001, 61571042 and 61671060), the Ministry Research Foundation, and the Ministry Key Laboratory Research Foundation.

7 References


Fig. 11 Comparison of ISAR imaging results of b727 airplane
(a) Method based on STFT in [14], (b) Method based on WVD in [16], (c) Method based on FrFT in [21], (d) Proposed method

Table 4 Performance evaluation by contrast of real dataset

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