Joint CSIT Acquisition Based on Low-Rank Matrix Recovery for FDD Massive MIMO Systems

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***Abstract*—Channel state information at the transmitter (CSIT) is essential for frequency-division duplexing (FDD) massive multiple-input multiple-output (MIMO) systems, but conventional solutions involve overwhelming overhead both for downlink channel training and uplink channel feedback. In this paper, we propose a joint CSIT acquisition scheme based on low-rank matrix recovery to reduce the overhead. Particularly, unlike conventional schemes where users individually estimate the channel and then feeds back the estimated CSI to the BS to realize CSIT, we propose that all scheduled users feed back their received pilots directly to the BS without individual channel estimation, and then joint MIMO channel estimation can be realized at the BS. We further formulate the joint channel estimation problem at the BS as a low-rank matrix recovery problem by utilizing the low-rank property of the massive MIMO channel matrix, which is caused by the limited number of clusters. Finally, we propose a hybrid low-rank matrix recovery algorithm based on the singular value projection to solve this problem, which can provide accurate CSIT with low overhead as demonstrated by simulations.**

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) equipped with tens or hundreds of transmit antennas at the base station (BS) has been widely recognized as a promising technology for future 5G wireless communication systems. In massive MIMO systems, the channel state information at the transmitter (CSIT) is essential for the BS to eliminate inter-user interference to simultaneously serve a set of users [1].

Conventional CSIT acquisition in frequency-division duplexing (FDD) MIMO systems consists of two separated steps: downlink channel training and uplink channel feedback. Specifically, the BS transmits orthogonal training pilots, and users individually employ least square (LS) or minimum mean square error (MMSE) algorithm to estimate the CSI [2]. After that, the estimated CSI is fed back to the BS through dedicated uplink channel to realize CSIT. For massive MIMO systems, as the number of BS antennas grows very large, downlink channel training overhead will be overwhelming [3]. Meanwhile, the large dimension of massive MIMO channel matrix also makes accurate CSI feedback be a challenging problem due to high overhead required for channel feedback [4].

In this paper, we propose a joint CSIT acquisition scheme based on low-rank matrix recovery for FDD massive MIMO systems, where the low-rank property of the massive MIMO channel matrix due to the limited number of clusters [5] is exploited to reduce the overhead for both downlink channel training and uplink channel feedback. Specifically, the BS first transmits pilots for downlink channel training, and then the scheduled users directly feed back their received pilots to the BS without individual channel estimation at the user side. Finally, joint CSIT is realized by joint channel estimation of all users at the BS, which is achieved by formulating the joint channel estimation problem as a low-rank matrix recovery problem. Particularly, to solve this problem, we propose the hybrid low-rank matrix recovery algorithm based on the singular value projection (SVP), which is able to provide good recovery accuracy and converge fast.

II. SYSTEM MODEL

We consider a massive MIMO system working in FDD mode with M antennas at the BS and K scheduled single-antenna users. The BS transmits pilots $\phi_t \in \mathcal{C}^{M\times 1}$ at the t-th channel use, where $t = 1, 2, \cdots, T$. At the k-th user, the received pilots $\mathbf{y}_k \in \mathcal{C}^{1\times T}$ can be expressed as

$$\mathbf{y}_k = \mathbf{h}_k\mathbf{\Phi} + \mathbf{n}_k^d, \quad (1)$$

where $\mathbf{\Phi} = [\phi_1, \phi_2, \cdots, \phi_T]$ of size $M \times T$ denotes the transmitted pilots during T channel uses, and $\mathbf{n}_k^d \in \mathcal{C}^{1\times T}$ represents the independent and identically distributed (i.i.d.) additive white complex Gaussian noise (AWGN), whose elements have zero mean and the variance $\sigma^2_{n_k^d}$. Channel vector $\mathbf{h}_k \in \mathcal{C}^{1\times M}$ between the BS and the k-th user is given by [5]

$$\mathbf{h}_k = \sum_{p=1}^{P} g_{k,p}\mathbf{a}(\theta_p), \quad (2)$$

where P is the number of resolvable physical paths between the BS and the user, $g_{k,p}$ denotes the propagation gain of the p-th path from the BS to the k-th user, the angle-of-departure (AoD) of the p-th path θ_p is limited to $\theta_p \in [-\pi/2, \pi/2]$, and we have $\mathbf{a}(\theta_p) = [1, e^{-j2\pi\frac{D}{\lambda}\cos(\theta_p)}, \cdots, e^{-j2\pi\frac{D}{\lambda}(M-1)\cos(\theta_p)}]$ for the widely used uniform linear arrays (ULA), where D and λ denote the distance between BS antennas and carrier wavelength, respectively.

III. PROPOSED JOINT CSIT ACQUISITION

Unlike conventional CSIT acquisition schemes, where $\mathbf{h}_k$ is estimated independently at different users and then fed back to the BS, we propose the joint CSIT acquisition scheme, where users directly feed back their received pilots $\mathbf{y}_k$ to the BS for joint MIMO channel estimation. As it has been demonstrated that the dedicated uplink channel for channel feedback can be modeled as an AWGN channel [6], the received pilot matrix at the BS $\mathbf{Y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \cdots, \mathbf{y}_K^T]^T \in \mathcal{C}^{K\times T}$ for all scheduled users can be expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{\Phi} + \mathbf{N}, \quad (3)$$

where $\mathbf{H} = [\mathbf{h}_1^T, \mathbf{h}_2^T, \cdots, \mathbf{h}_K^T]^T \in \mathcal{C}^{K\times M}$ is the MIMO channel matrix, and $\mathbf{N} = [\mathbf{n}_1^T, \mathbf{n}_2^T, \cdots, \mathbf{n}_K^T]^T$ is the noise matrix, whose entries have the variance σ_n^2 denoting the overall

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noise power in the downlink and uplink. Based on (2), we have $\mathbf{H} = \mathbf{G}\mathbf{A}$, where $\mathbf{G} \in \mathcal{C}^{K\times P}$ with the (k,p)-th entry being $g_{k,p}$, and $\mathbf{A} = [\mathbf{a}(\theta_1)^T, \mathbf{a}(\theta_2)^T, \cdots, \mathbf{a}(\theta_P)^T]^T \in \mathcal{C}^{P\times M}$. As $\text{rank}(\mathbf{H}) \leq \min\{\text{rank}(\mathbf{G}), \text{rank}(\mathbf{A})\}$, we have $\text{rank}(\mathbf{H}) \leq \min\{M, K, P\}$. For massive MIMO systems, we usually have large M and K, while P is relatively small due to the limited number of active clusters [1], [5]. Thus, we have $\text{rank}(\mathbf{H}) \leq P$, which means the rank of $\mathbf{H}$ is much smaller than the dimension of $\mathbf{H}$.

By exploiting the low-rank property of $\mathbf{H}$, we can formulate the joint MIMO channel estimation problem as a low-rank matrix recovery problem:

$$\hat{\mathbf{H}} = \underset{\mathbf{H},\ \text{rank}(\mathbf{H})\leq P}{\arg\min}\ J(\mathbf{H}) = ||\mathbf{Y} - \mathbf{H}\mathbf{\Phi})||_F^2. \tag{4}$$

Without the low-rank constraint $\text{rank}(\mathbf{H}) \leq P$, conventional gradient descent algorithm and Newton's algorithm can achieve the optimal solution to the unconstrained optimization problem $\arg\min J(\mathbf{H}) = ||\mathbf{Y} - \mathbf{H}\mathbf{\Phi})||_F^2$. To solve (4), SVP-based gradient decent algorithm (SVP-G) and SVP-based Newton's algorithm (SVP-N) [7] have been proposed to make the solution satisfy the low-rank constraint $\text{rank}(\mathbf{H}) \leq P$ by projecting $\mathbf{H}^{(i)}$ onto a low-rank matrix $\mathbf{H}_q^{(i)}$ in each iteration via $\text{svp}(\mathbf{H}^{(i)})$ operation, which is defined as $\text{svp}(\mathbf{H}) = \sum_{r=1}^{q} \mathbf{u}_r\sigma_r\mathbf{v}_r^T$. However, as the cost function $J(\mathbf{H})$ is quadratic convex, SVP-G converges after one iteration [7], which results in a poor solution because $\text{svp}(\cdot)$ will be executed only once, i.e., the low-rank constraint will be only used once. On the other hand, SVP-G executes $\text{svp}(\cdot)$ in every iteration to achieve a more accurate solution, but it converges slowly. To combine both advantages of SVP-G and SVP-N, we propose the SVP-based hybrid low-rank matrix recovery algorithm (SVP-H) as shown in **Algorithm 1**, where SVP-N is used in the first iteration (see step 4) to realize fast convergence, while SVP-G is used in the rest iterations (see step 6) to achieve high accuracy. Note that the initialization matrix $\mathbf{R}$ is randomly generated whose elements follow i.i.d. $\mathcal{N}(0, 1)$.

Algorithm 1 The proposed SVP-H algorithm

Input: $\mathbf{y}$; $\mathbf{\Psi}$; q
Output: $\hat{\mathbf{H}}$
1: Initialization : $\mathbf{H}^{(0)} \leftarrow \mathbf{R}$, $\mathbf{h}^{(0)} \leftarrow \text{vec}(\mathbf{H}^{(0)})$, $\mathbf{H}_q^{(0)} \leftarrow \text{svp}(\mathbf{H}^{(0)})$, $\mathbf{h}_q^{(0)} \leftarrow \text{vec}(\mathbf{H}_q^{(0)})$, $i \leftarrow 1$.
2: **while** $i \leq i_{\max}$ **do**
3: **if** $i = 1$ **then**
4: $\lambda^{(i)} \leftarrow -s$, $\mathbf{d}^{(i)} \leftarrow \nabla J(\text{vec}(\mathbf{H}_q^{(i-1)}))$ % SVP-N
5: **else**
6: $\lambda^{(i)} \leftarrow -1$, $\mathbf{d}^{(i)} \leftarrow \nabla^2 J(\text{vec}(\mathbf{H}_q^{(i-1)}))^{-1} \times \nabla J(\text{vec}(\mathbf{H}_q^{(i-1)}))$ % SVP-G
7: **end if**
8: $\mathbf{h}^{(i)} \leftarrow \mathbf{h}_q^{(i-1)} + \lambda^{(i)}\mathbf{d}^{(i)}$, $\mathbf{H}^{(i)} \leftarrow \text{unvec}(\mathbf{h}^{(i)})$
9: $\mathbf{H}_q^{(i)} \leftarrow \text{svp}(\mathbf{H}^{(i)})$, $\mathbf{h}_q^{(i)} \leftarrow \text{vec}(\mathbf{H}_q^{(i)})$
10: $i \leftarrow i + 1$
11: **end while**
12: **return** $\hat{\mathbf{H}} \leftarrow \mathbf{H}_q^{(i)}$

IV. Simulation Results

Simulation parameters are set as follows: $M = 60$, $K = 40$, $P = 20$, $T = 80$; $\frac{D}{\lambda} = 0.3$, $\theta_p = -\pi/2 + \frac{p-1}{P}\pi$; $i_{\max} = 150$, $q = 11$, $s = 0.002$. Fig. 1 shows the normalized mean squared error (NMSE) performance comparison between the conventional CSIT acquisition and the proposed joint CSIT acquisition. Note that LS is used for channel estimation at users for conventional scheme. It is clear that the proposed scheme has much smaller NMSE than the conventional one with the same training and feedback overhead. In other words, to achieve the same NMSE, the proposed scheme requires less overhead than the conventional one.

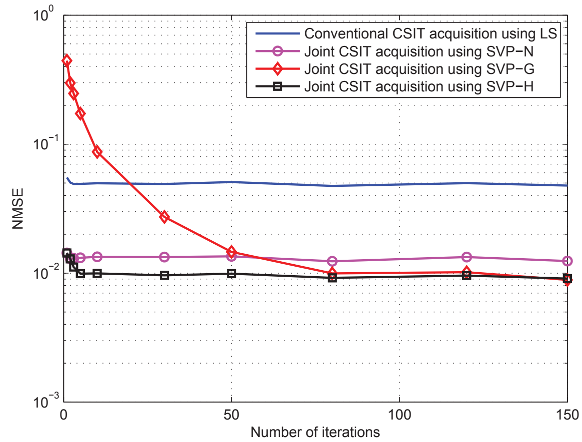

Fig. 1. NMSE comparison with total SNR=20 dB.

V. Conclusion

In this paper, we investigate the challenging problem of CSIT acquisition in FDD massive MIMO systems. We utilize the low-rank property of MIMO channel matrix to propose a joint CSIT acquisition scheme and formulate it as a low-rank matrix recovery problem. We further propose the SVP-H algorithm to solve this problem, which can provide accurate CSIT with low overhead.

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