Optimal FemtoCell density for Maximizing Throughput in 5G Heterogeneous Networks Under Outage Constraints

Talha Mir[†], Linglong Dai[†], Yang Yang[‡†], Wenqian Shen[†], and Bichai Wang[†]
[†]Tsinghua National Laboratory for Information Science and Technology (TNList),
Department of Electronic Engineering, Tsinghua University, Beijing 100084, China
[‡]School of Information and Communication Engineering, Beijing University of Posts and Telecommunications, Beijing, China
Email: {bah15, swq13, wbc15}@mails.tsinghua.edu.cn, {daill, yy0553}@tsinghua.edu.cn

***Abstract*—Heterogeneous networks (HetNets), which involve densely deployed femtocells underlaid traditional macrocell network, is a promising solution to the extremely high data rate requirements of the future 5G communications. In this paper, we analyze the closed-form optimal deployment of femtocells in HetNets to maximize the network throughput under the outage constraints from both macrocells and femtocells. Specifically, we model the random distribution of macro cell users (MUEs) and femtocell base stations (FBSs) as Poisson Point Processes (PPPs). Then, the closed form expressions for outage probabilities in both uplink and downlink transmissions are derived. Further, we study the network throughput maximization problem under the outage probability constraints. Finally, With the help of convex optimization, the interval of FBS density, which contains the maximum network throughput is obtained in closed form. Simulation results validate the impact of the system parameters on the different optimal FBS density as well as the influence of interference to the maximum network throughput.**

***Index Terms*—Heterogenous networks, network throughput, stochastic geometry, convex optimization.**

I. Introduction

The vision of future 5G wireless communications lies in providing very high data rate, significant improvement in users perceived quality of services(QoS), manifold increase in base station capacity, and extremely low latency compared to current 4G LTE networks [1]. However, the explosion of mobile subscribers and service demands lead to a heavy overload for traditional macro base stations (MBSs) [2]. Heterogeneous networks (HetNets), where various femtocells can be deployed in the traditional macrocell network, has been considered as a paradigm shift to offload over-burdened MBSs and facilitate higher network throughput with better spectral efficiency. On one hand, the increasing network density can substantially improve the network capacity. On the other hand, densely deployed femtocells in co-channel mode cause severe interference and thus the obvious performance degradation of the macrocell network, which is the key challenge for the deployment of femtocells in HetNets [3].

However, interference mitigation in such networks remains an open problem. In order to reduce such interference, some solutions have been proposed to enhance the network performance, where the transmission rate is a key performance metric of HetNets. Specifically, based on the interference statistics between macro and femtocells, the system performance of HetNet was optimized by a coalition algorithm in a single macro cell. Further, cooperative spectrum sharing is considered, the optimization of transmission rate for femtocells in a single macro cell is investigated in [4]. Then, the analysis of transmission rate was extended with the consideration of quality-of-service (QoS) requirement in [5]. Where all analysis are based on one or multiple femto-scale macro cells. However, for practical HetNets, it is necessary to analyze the transmission rate of large-scale networks with the random deployment of MUEs and FBSs, as well the interference consideration from inter and intra cells.

In this paper by considering both, the network performance optimization, and random distribution of users and FBSs in HetNets, the statistics of interference are analyzed through stochastic geometry, and the network throughput is maximized by an optimal FBSs deployment. The main contributions of paper can be summarized as follows:

- With the help of Laplace transformation, the outage probability at the MBS and femtocells (both uplink and downlink) are derived in closed form.
- We formulate the network throughput maximization problem of femtocells under the outage probability constraints, and derive the optimal FBS density to maximize the network throughput. The optimal FBS density is obtained in closed from for this non-convex optimization problem.
- Simulation results discuss the outage probability and network throughput of femtocells under different system parameters.

The rest of the paper is organized as follows. Section II describes the system model. Section III presents the outage probabilities and the network throughput for femtocells. Section IV shows the optimization analysis of FBS density for maximizing network throughput. Simulation results are discussed in Section V. Finally, our conclusions are summarized

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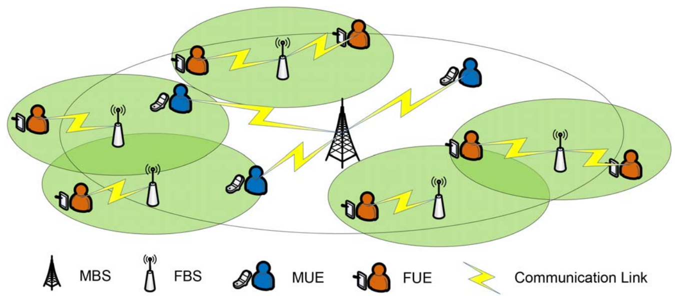

Fig. 1. The scenario of heterogeneous networks

in Section VI.

II. System Model

As shown in Fig. 1, we consider the HetNets where femtocells are underlaid the traditional macrocell network. The uplink resources of the macrocell network are shared by femtocells. We model the macro cell users (MUEs) as an independent homogeneous Poisson Point Process (PPP) $\Pi_{\rm M}$ on two dimensional plane $\Re$, with density $\lambda_{\rm M}$. Femtocell base stations (FBS) also satisfy an independent homogeneous PPP on $\Re$, denoted as $\Pi_{\rm F}$, with density $\lambda_{\rm F}$. The traffic in the HetNets is assumed as full buffer. The powers for MUEs, FBSs and FUEs are defined as $P_{\rm M}$, $P_{\rm Fd}$ and $P_{\rm Fu}$, respectively. Defining uplink and down transmission probabilities in femtocells as ${\rm Pr_u}$, $(0 \le {\rm Pr_u} \le 1)$, and ${\rm Pr_d}$ as ${\rm Pr_d} = 1 - {\rm Pr_u}$, $(0 \le {\rm Pr_d} \le 1)$ respectively. According to Slivnyak's theorem [6], a typical receiver of femtocells is placed at the origin of plane $\Re$. While when we consider the downlink transmission this receiver is an FUE, otherwise it is an FBS in uplink transmission.

We consider following propagation model in HetNets:

$$P_{\rm r} = P_{\rm t}\epsilon_{\rm t,r} D_{\rm t,r}^{-\alpha}, \tag{1}$$

where P_t and P_r represent the transmitter and receiver powers respectively. $D_{\rm t,r}$ is the distance, and α denotes path loss exponent which satisfies $\alpha \ge 2$. $\epsilon_{\rm t,r}$ denotes the Rayleigh fading coefficient following an independent exponential distribution with unit mean for every communication link in the HetNets.

III. Network Throughput of Femtocells

In this section, we calculate the outage probabilities in HetNets, and then the network throughput for femtocells is obtained.

A. Outage Probabilities in HetNets

The signal-to-interference ratio (SIR) at a typical MBS is

$$\gamma_{\rm M} = \frac{P_{\rm M}\epsilon_{\rm M0} D_{\rm M0}^{-\alpha}}{\sum\limits_{k\in\Pi_{\rm M}} \frac{P_{\rm M}\epsilon_{k0}}{D_{k0}^{\alpha}} + \sum\limits_{i\in(\Pi_{\rm F}\cap UP)} \frac{P_{\rm Fu}\epsilon_{i0}}{D_{i0}^{\alpha}} + \sum\limits_{i\in(\Pi_{\rm F}\cap DW)} \frac{P_{\rm Fd}\epsilon_{i0}}{D_{i0}^{\alpha}}}, \tag{2}$$

where $\epsilon_{\rm M0}$ and $D_{\rm M0}$ denote the Rayleigh fading coefficient and the distance from the desired MUE to the typical MBS. Similarly, ϵ_{k0} and D_{k0} represent the Rayleigh fading coefficient and the distance from node k to the origin in macrocell network, while ϵ_{i0} and D_{i0} are parameters for node i in femtocells. Compact point sets as UP and DW satisfy, $UP = \{x|$ node x is the receiver in uplink transmission in femtocells$\}$, and $DW = \{y|$node y is the receiver in downlink transmission in femtocells$\}$, respectively.

Let $I_{\rm M} = \sum\limits_{k\in\Pi_{\rm M}} \epsilon_{k0} D_{k0}^{-\alpha}$, $I_{\rm Fu} = \sum\limits_{i\in(\Pi_{\rm F}\cap UP)} \frac{P_{\rm Fu}}{P_{\rm M}}\epsilon_{i0} D_{i0}^{-\alpha}$, and $I_{\rm Fd} = \sum\limits_{i\in(\Pi_{\rm F}\cap DW)} \frac{P_{\rm Fd}}{P_{\rm M}}\epsilon_{i0} D_{i0}^{-\alpha}$, (2) can be simplified as

$$\gamma_{\rm M} = \frac{\epsilon_{\rm M0} D_{\rm M0}^{-\alpha}}{I_{\rm M} + I_{\rm Fu} + I_{\rm Fd}}, \tag{3}$$

Then, the following lemma shows the outage probability of typical MBS in HetNets:

Lemma 1. *The outage probability of a typical MBS in HetNets must satisfy*

$$\Pr(\gamma_{\rm M} < \zeta_{\rm M}) = 1 - e^{-\lambda_{\rm M}\mu_{\rm M} - \lambda_{\rm F}\mu_{\rm M}\left[{\rm Pr_u}\left(\frac{P_{\rm Fu}}{P_{\rm M}}\right)^{\frac{2}{\alpha}} + {\rm Pr_d}\left(\frac{P_{\rm Fd}}{P_{\rm M}}\right)^{\frac{2}{\alpha}}\right]}, \tag{4}$$

where $\Pr(\cdot)$ *denote the probability,* $\zeta_{\rm M}$ *is the SIR threshold of uplink transmission for MUE,* $\mu_{\rm M} = \pi D_{\rm M0}^2 \zeta_{\rm M}^{\frac{2}{\alpha}} \Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)$*, and* $\Gamma(\cdot)$ *represents the gamma function with the form* $\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt$*.*

Proof: From (3) we have

$$\Pr(\gamma_{\rm M} < \zeta_{\rm M}) = \Pr\left[\epsilon_{\rm M0} < \zeta_{\rm M} D_{\rm M0}^{\alpha}(I_{\rm M} + I_{\rm Fu} + I_{\rm Fd})\right], \tag{5}$$

As $\zeta_{\rm M}$ satisfies the independent exponential distribution with unit mean, (5) can be written as

$$\begin{aligned} &\Pr(\gamma_{\rm M} < \zeta_{\rm M}) = 1 - E\left[e^{-\zeta_{\rm M} D_{\rm M0}^{\alpha}(I_{\rm M}+I_{\rm Fu}+I_{\rm Fd})}\right] \\ &= 1 - \mathcal{L}_{I_{\rm M}}(\zeta_{\rm M} D_{\rm M0}^{\alpha})\mathcal{L}_{I_{\rm Fu}}(\zeta_{\rm M} D_{\rm M0}^{\alpha})\mathcal{L}_{I_{\rm Fd}}(\xi_{\rm M} D_{\rm M0}^{\alpha}), \end{aligned} \tag{6}$$

where $\mathcal{L}_{I_{\rm M}}(\cdot)$, $\mathcal{L}_{I_{\rm Fu}}(\cdot)$, and $\mathcal{L}_{I_{\rm Fd}}(\cdot)$ are Laplace transformation of $I_{\rm M}$, $I_{\rm Fu}$, and $I_{\rm Fd}$, respectively

$$\begin{aligned} &\mathcal{L}_{I_{\rm M}}(\zeta_{\rm M} D_{\rm M0}^{\alpha}) = \exp\left[-\lambda_{\rm M}\int_0^\infty E_{\epsilon_{k0}}\left(1 - e^{-\zeta_{\rm M} D_{\rm M0}^{\alpha} r^{-\alpha}}\right)dr\right] \\ &= \exp\left[-\lambda_{\rm M}\pi D_{00}^2 \zeta_{\rm M}^{\frac{2}{\alpha}}\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\right], \end{aligned} \tag{7}$$

$$\begin{aligned} &\mathcal{L}_{I_{\rm Fu}}(\zeta_{\rm M} D_{\rm M0}^{\alpha}) \\ &= \exp\left[-\lambda_{\rm F}{\rm Pr_u}\left(\frac{P_{\rm Fu}}{P_{\rm M}}\right)^{\frac{2}{\alpha}}\pi D_{00}^2\zeta_{\rm M}^{\frac{2}{\alpha}}\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\right], \end{aligned} \tag{8}$$

$$\begin{aligned} &\mathcal{L}_{I_{\rm Fd}}(\zeta_{\rm M} D_{\rm M0}^{\alpha}) \\ &= \exp\left[-\lambda_{\rm F}{\rm Pr_d}\left(\frac{P_{\rm Fd}}{P_{\rm M}}\right)^{\frac{2}{\alpha}}\pi D_{00}^2\zeta_{\rm M}^{\frac{2}{\alpha}}\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\right], \end{aligned} \tag{9}$$

By denote $\mu_{\rm M} = \pi D_{\rm M0}^2\zeta_{\rm M}^{\frac{2}{\alpha}}\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)$, (4) can be obtained. ∎

Similarly, the SIR at a typical FUE and FBS in uplink and downlink satisfies

$$\gamma_{\rm Fu} = \frac{P_{\rm Fu}\epsilon_{\rm Fu0} D_{\rm Fu0}^{-\alpha}}{\sum\limits_{k\in\Pi_{\rm M}} \frac{P_{\rm M}\epsilon_{k0}}{D_{k0}^{\alpha}} + \sum\limits_{i\in(\Pi_{\rm F}\cap UP)} \frac{P_{\rm Fu}\epsilon_{i0}}{D_{i0}^{\alpha}} + \sum\limits_{i\in(\Pi_{\rm F}\cap DW)} \frac{P_{\rm Fd}\epsilon_{i0}}{D_{i0}^{\alpha}}}, \tag{10}$$

$$\gamma_{\rm Fd} = \frac{P_{\rm Fd}\epsilon_{\rm Fd0} D_{\rm Fd0}^{-\alpha}}{\sum\limits_{k\in\Pi_{\rm M}} \frac{P_{\rm M}\epsilon_{k0}}{D_{k0}^{\alpha}} + \sum\limits_{i\in(\Pi_{\rm F}\cap UP)} \frac{P_{\rm Fu}\epsilon_{i0}}{D_{i0}^{\alpha}} + \sum\limits_{i\in(\Pi_{\rm F}\cap DW)} \frac{P_{\rm Fd}\epsilon_{i0}}{D_{i0}^{\alpha}}}, \tag{11}$$

Denoting ζ_{Fu} and ζ_{Fd} as SIR thresholds for uplink and downlink transmissions in femtocells, and following the same proof steps as in **Lemma 1**, we have:

Lemma 2. *The outage probability of the typical FUE in uplink transmission satisfies*

$$\begin{aligned}&\Pr\left(\gamma_{\mathrm{Fu}}<\zeta_{\mathrm{Fu}}\right)\\&=1-e^{-\lambda_{\mathrm{M}}\mu_{\mathrm{Fu}}\left(\frac{P_{\mathrm{M}}}{P_{\mathrm{Fu}}}\right)^{\frac{2}{\alpha}}-\lambda_{\mathrm{F}}\mu_{\mathrm{Fu}}\left[\mathrm{Pr_u}+\mathrm{Pr_d}\left(\frac{P_{\mathrm{Fd}}}{P_{\mathrm{Fu}}}\right)^{\frac{2}{\alpha}}\right]},\end{aligned}\tag{12}$$

where $\mu_{\mathrm{Fu}}=\pi D_{\mathrm{Fu}0}^2\zeta_{\mathrm{Fu}}^{\frac{2}{\alpha}}\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)$.

Lemma 3. *The outage probability of typical FBS in downlink transmission satisfies*

$$\begin{aligned}&\Pr\left(\gamma_{\mathrm{Fd}}<\zeta_{\mathrm{Fd}}\right)\\&=1-e^{-\lambda_{\mathrm{M}}\mu_{\mathrm{Fd}}\left(\frac{P_{\mathrm{M}}}{P_{\mathrm{Fd}}}\right)^{\frac{2}{\alpha}}-\lambda_{\mathrm{F}}\mu_{\mathrm{Fd}}\left[\mathrm{Pr_u}\left(\frac{P_{\mathrm{Fu}}}{P_{\mathrm{Fd}}}\right)^{\frac{2}{\alpha}}+\mathrm{Pr_d}\right]},\end{aligned}\tag{13}$$

where $\mu_{\mathrm{Fd}}=\pi R_{Sd0}^2\xi_{\mathrm{Fd}}^{\frac{2}{\alpha}}\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)$.

B. Network Throughputs for FemtoCells

In HetNets, the network throughput of femtocells is defined as [7]

$$T_{Fn}(\lambda_F)=\lambda_F\mathrm{Pr}_n\overline{R}_{Fn},n\in(u,d),\tag{14}$$

Where $\overline{R_{\mathrm{F}n}}$ satisfies the following form [8]

$$\overline{R_{\mathrm{F}n}}=\sup_{\zeta_{\mathrm{F}n}\geq 0}W\log_2\left(1+\zeta_{\mathrm{F}n}\right)\{1-\Pr\left(\gamma_{\mathrm{F}n}<\zeta_{\mathrm{F}n}\right)\},\tag{15}$$

Definition 1. *The network throughput for uplink transmission in femtocells is*

$$\begin{aligned}T_{\mathrm{Fu}}\left(\lambda_{\mathrm{F}}\right)=&W\mathrm{Pr_u}\lambda_{\mathrm{F}}\log_2\left(1+\zeta_{\mathrm{Fu}}\right)\\&\times e^{-\lambda_{\mathrm{M}}\mu_{\mathrm{Fu}}\left(\frac{P_{\mathrm{M}}}{P_{\mathrm{Fu}}}\right)^{\frac{2}{\alpha}}-\lambda_{\mathrm{F}}\mu_{\mathrm{Fu}}\left[\mathrm{Pr_u}+\mathrm{Pr_d}\left(\frac{P_{\mathrm{Fd}}}{P_{\mathrm{Fu}}}\right)^{\frac{2}{\alpha}}\right]},\end{aligned}\tag{16}$$

The network throughput for downlink transmission in femtocells is

$$\begin{aligned}T_{\mathrm{Fd}}\left(\lambda_{\mathrm{F}}\right)=&W\mathrm{Pr_d}\lambda_{\mathrm{F}}\log_2\left(1+\zeta_{\mathrm{Fd}}\right)\\&\times e^{-\lambda_{\mathrm{M}}\mu_{\mathrm{Fd}}\left(\frac{P_{\mathrm{M}}}{P_{\mathrm{Fd}}}\right)^{\frac{2}{\alpha}}-\lambda_{\mathrm{F}}\mu_{\mathrm{Fd}}\left[\mathrm{Pr_u}\left(\frac{P_{\mathrm{Fu}}}{P_{\mathrm{Fd}}}\right)^{\frac{2}{\alpha}}+\mathrm{Pr_d}\right]},\end{aligned}\tag{17}$$

Thus, network throughput for femtocells $T\left(\lambda_{\mathrm{F}}\right)$ satisfies

$$T\left(\lambda_{\mathrm{F}}\right)=T_{\mathrm{Fu}}\left(\lambda_{\mathrm{F}}\right)+T_{\mathrm{Fd}}\left(\lambda_{\mathrm{F}}\right),\tag{18}$$

IV. Optimization of the Femtocells Density

When femtocells reuse the frequency resources of macrocell network, the reliable transmission of macrocell should be guaranteed. Considering uplink and downlink transmissions in femtocells, we have the following four constraints:

$$0\leq\lambda_{\mathrm{F}}\leq\lambda_{F,M},\tag{19}$$

$$1-e^{-\lambda_{\mathrm{M}}\mu_{\mathrm{M}}-\lambda_{\mathrm{F}}\mu_{\mathrm{M}}\left[\mathrm{Pr_u}\left(\frac{P_{\mathrm{Fu}}}{P_{\mathrm{M}}}\right)^{\frac{2}{\alpha}}+\mathrm{Pr_d}\left(\frac{P_{\mathrm{Fd}}}{P_{\mathrm{M}}}\right)^{\frac{2}{\alpha}}\right]}\leq\phi_{\mathrm{M}},\tag{20}$$

$$1-e^{-\lambda_{\mathrm{M}}\mu_{\mathrm{Fu}}\left(\frac{P_{\mathrm{M}}}{P_{\mathrm{Fu}}}\right)^{\frac{2}{\alpha}}-\lambda_{\mathrm{F}}\mu_{\mathrm{Fu}}\left[\mathrm{Pr_u}+\mathrm{Pr_d}\left(\frac{P_{\mathrm{Fd}}}{P_{\mathrm{Fu}}}\right)^{\frac{2}{\alpha}}\right]}\leq\phi_{\mathrm{Fu}},\tag{21}$$

$$1-e^{-\lambda_{\mathrm{M}}\mu_{\mathrm{Fd}}\left(\frac{P_{\mathrm{M}}}{P_{\mathrm{Fd}}}\right)^{\frac{2}{\alpha}}-\lambda_{\mathrm{F}}\mu_{\mathrm{Fd}}\left[\mathrm{Pr_u}\left(\frac{P_{\mathrm{Fu}}}{P_{\mathrm{Fd}}}\right)^{\frac{2}{\alpha}}+\mathrm{Pr_d}\right]}\leq\phi_{\mathrm{Fd}},\tag{22}$$

where $\lambda_{F,M}$ is the maximum density of FBS in HetNets. ϕ_{M}, ϕ_{Fu}, and ϕ_{Fd} are the outage probability thresholds of MBS, FBS and FUEs, respectively. We get the following network throughput maximization problem for femtocells:

$$\begin{aligned}\max\quad & T\left(\lambda_{\mathrm{F}}\right)\\ s.t.\quad & (19)-(22).\end{aligned}\tag{23}$$

From above inequalities in (20)-(22), we have

$$\lambda_{\mathrm{F}}\leq\frac{-\lambda_{\mathrm{M}}P_{\mathrm{M}}^{\frac{2}{\alpha}}-\frac{P_{\mathrm{M}}^{\frac{2}{\alpha}}}{\mu_{\mathrm{M}}}\ln\left(1-\phi_{\mathrm{M}}\right)}{\mathrm{Pr_u}P_{\mathrm{Fu}}^{\frac{2}{\alpha}}+\mathrm{Pr_d}P_{\mathrm{Fd}}^{\frac{2}{\alpha}}},\tag{24}$$

$$\lambda_{\mathrm{F}}\leq\frac{-\lambda_{\mathrm{M}}P_{\mathrm{M}}^{\frac{2}{\alpha}}-\frac{P_{\mathrm{Fu}}^{\frac{2}{\alpha}}}{\mu_{\mathrm{Fu}}}\ln\left(1-\phi_{\mathrm{Fu}}\right)}{\mathrm{Pr_u}P_{\mathrm{Fu}}^{\frac{2}{\alpha}}+\mathrm{Pr_d}P_{\mathrm{Fd}}^{\frac{2}{\alpha}}},\tag{25}$$

$$\lambda_{\mathrm{F}}\leq\frac{-\lambda_{\mathrm{M}}P_{\mathrm{M}}^{\frac{2}{\alpha}}-\frac{P_{\mathrm{Fd}}^{\frac{2}{\alpha}}}{\mu_{\mathrm{Fd}}}\ln\left(1-\phi_{\mathrm{Fd}}\right)}{\mathrm{Pr_u}P_{\mathrm{Fu}}^{\frac{2}{\alpha}}+\mathrm{Pr_d}P_{\mathrm{Fd}}^{\frac{2}{\alpha}}},\tag{26}$$

Define $\lambda_{\mathrm{F,sup}\,1}=\frac{-\lambda_{\mathrm{M}}P_{\mathrm{M}}^{\frac{2}{\alpha}}-\frac{P_{\mathrm{M}}^{\frac{2}{\alpha}}}{\mu_{\mathrm{M}}}\ln(1-\phi_{\mathrm{M}})}{\mathrm{Pr_u}P_{\mathrm{Fu}}^{\frac{2}{\alpha}}+\mathrm{Pr_d}P_{\mathrm{Fd}}^{\frac{2}{\alpha}}}$, $\lambda_{\mathrm{F,sup}\,2}=\frac{-\lambda_{\mathrm{M}}P_{\mathrm{M}}^{\frac{2}{\alpha}}-\frac{P_{\mathrm{Fu}}^{\frac{2}{\alpha}}}{\mu_{\mathrm{Fu}}}\ln(1-\phi_{\mathrm{Fu}})}{\mathrm{Pr_u}P_{\mathrm{Fu}}^{\frac{2}{\alpha}}+\mathrm{Pr_d}P_{\mathrm{Fd}}^{\frac{2}{\alpha}}}$, $\lambda_{\mathrm{F,sup}\,3}=\frac{-\lambda_{\mathrm{M}}P_{\mathrm{M}}^{\frac{2}{\alpha}}-\frac{P_{\mathrm{Fd}}^{\frac{2}{\alpha}}}{\mu_{\mathrm{Fd}}}\ln(1-\phi_{\mathrm{Fd}})}{\mathrm{Pr_u}P_{\mathrm{Fu}}^{\frac{2}{\alpha}}+\mathrm{Pr_d}P_{\mathrm{Fd}}^{\frac{2}{\alpha}}}$. Then, $\lambda_{\mathrm{F,sup}}=\{\lambda_{\mathrm{F,max}},\lambda_{\mathrm{F,sup}\,1},\lambda_{\mathrm{F,sup}\,2},\lambda_{\mathrm{F,sup}\,3}\}$.

Let $X_1=W\mathrm{Pr_u}\log_2\left(1+\zeta_{\mathrm{Fu}}\right)$, $X_2=\lambda_{\mathrm{M}}\mu_{\mathrm{Fu}}\left(\frac{P_{\mathrm{M}}}{P_{\mathrm{Fu}}}\right)^{\frac{2}{\alpha}}$, $X_3=\mu_{\mathrm{Fu}}\left[\mathrm{Pr_u}+\mathrm{Pr_d}\left(\frac{P_{\mathrm{Fd}}}{P_{\mathrm{Fu}}}\right)^{\frac{2}{\alpha}}\right]$, $Y_1=W\mathrm{Pr_d}\log_2\left(1+\zeta_{\mathrm{Fd}}\right)$, $Y_2=\lambda_{\mathrm{M}}\mu_{\mathrm{Fd}}\left(\frac{P_{\mathrm{M}}}{P_{\mathrm{Fd}}}\right)^{\frac{2}{\alpha}}$, $Y_3=\mu_{\mathrm{Fd}}\left[\mathrm{Pr_u}\left(\frac{P_{\mathrm{Fu}}}{P_{\mathrm{Fd}}}\right)^{\frac{2}{\alpha}}+\mathrm{Pr_d}\right]$. Then, we have following theorem

Theorem 1. *The maximum value of function* $T\left(\lambda_{\mathrm{F}}\right)$ *locates at the interval* $[\lambda_{F,min},\lambda_{F,max}]$, *where* $\lambda_{F,min}=\min\left\{\frac{1}{X_3},\frac{1}{Y_3}\right\}$ *and* $\lambda_{F,max}=\max\left\{\frac{1}{X_3},\frac{1}{Y_3}\right\}$.

Proof: According to (16) and (17), we have $T_{\mathrm{Fu}}\left(\lambda_{\mathrm{F}}\right)=X_1\lambda_{\mathrm{F}}e^{-X_2-\lambda_{\mathrm{F}}X_3}$ and $T_{\mathrm{Fd}}\left(\lambda_{\mathrm{F}}\right)=Y_1\lambda_{\mathrm{F}}e^{-Y_2-\lambda_{\mathrm{F}}Y_3}$. Take the first order derivative of both $T_{\mathrm{Fu}}\left(\lambda_{\mathrm{F}}\right)$ and $T_{\mathrm{Fd}}\left(\lambda_{\mathrm{F}}\right)$, we have

$$T_{\mathrm{Fu}}'\left(\lambda_{\mathrm{F}}\right)\big|_{\lambda_{\mathrm{F}}}=X_1e^{-X_2-\lambda_{\mathrm{F}}X_3}\left(1-\lambda_{\mathrm{F}}X_3\right).\tag{27}$$

$$T_{\mathrm{Fd}}'\left(\lambda_{\mathrm{F}}\right)\big|_{\lambda_{\mathrm{F}}}=X_1e^{-X_2-\lambda_{\mathrm{F}}X_3}\left(1-\lambda_{\mathrm{F}}X_3\right).\tag{28}$$

Make $T_{\mathrm{Fu}}'\left(\lambda_{\mathrm{F}}\right)\big|_{\lambda_{\mathrm{F}}}=0$ and $T_{\mathrm{Fd}}'\left(\lambda_{\mathrm{F}}\right)\big|_{\lambda_{\mathrm{F}}}=0$, we get the maximum values of function $T_{\mathrm{Fu}}\left(\lambda_{\mathrm{F}}\right)$ and $T_{\mathrm{Fd}}\left(\lambda_{\mathrm{F}}\right)$ as $\frac{1}{X_3}$ and $\frac{1}{Y_3}$, respectively. When $\lambda_{\mathrm{F}}\in\left[0,\frac{1}{X_3}\right)$, the function $T_{\mathrm{Fu}}\left(\lambda_{\mathrm{F}}\right)$ increases monotonically, and $T_{\mathrm{Fu}}\left(\lambda_{\mathrm{F}}\right)$ decreases monotonically when $\lambda_{\mathrm{F}}\in\left(\frac{1}{X_3},+\infty\right)$. Similarly, the function $T_{\mathrm{Fd}}\left(\lambda_{\mathrm{F}}\right)$ increases monotonically when $\lambda_{\mathrm{F}}\in\left[0,\frac{1}{Y_3}\right)$, and it decreases monotonically with $\lambda_{\mathrm{F}}\in\left(\frac{1}{Y_3},+\infty\right)$.

We denote $\lambda_{F,min} = \min\left\{\frac{1}{X_3}, \frac{1}{Y_3}\right\}$, $\lambda_{F,max} = \max\left\{\frac{1}{X_3}, \frac{1}{Y_3}\right\}$. Suppose $\lambda_{F,min} = \frac{1}{X_3}$, and $\lambda_{F,max} = \frac{1}{Y_3}$. Then, for $\forall\lambda_{\mathrm{F}} \in \left[0, \frac{1}{X_3}\right)$, it can be known that, $T_{\mathrm{Fu}}\left(\lambda_{\mathrm{F}}\right) < T_{\mathrm{Fu}}\left(\frac{1}{X_3}\right)$ and $T_{\mathrm{Fd}}\left(\lambda_{\mathrm{F}}\right) < T_{\mathrm{Fd}}\left(\frac{1}{X_3}\right)$, because $\lambda_{\mathrm{F}} < \frac{1}{X_3} < \frac{1}{Y_3}$. Similarly, for $\forall\lambda_{\mathrm{F}} \in \left(\frac{1}{Y_3}, +\infty\right)$,

$$T\left(\lambda_{\mathrm{F}}\right) < T\left(\frac{1}{X_3}\right), \quad \lambda_{\mathrm{F}} \in \left(\frac{1}{Y_3}, +\infty\right). \tag{29}$$

The same results can be obtained if we define $\lambda_{F,min} = \frac{1}{Y_3}$ and $\lambda_{F,max} = \frac{1}{X_3}$. Therefore, the maximum value of function $T\left(\lambda_{\mathrm{F}}\right)$ must locate between the interval $[\lambda_{F,min}, \lambda_{F,max}]$. ■

Then, we have following theorem

Theorem 2. *The optimal FBS density $\lambda_{\mathrm{F}}^{opt}$ for maximizing the network throughput of femtocells satisfies*

$$\lambda_{\mathrm{F}}^{opt} = \begin{cases} \lambda_{\mathrm{F,sup}}, & \lambda_{\mathrm{F,sup}} < \lambda_{\mathrm{F}}^{*}, \\ \lambda_{\mathrm{F}}^{*}, & \lambda_{\mathrm{F}}^{*} \le \lambda_{\mathrm{F\,sup}}, \end{cases} \tag{30}$$

where λ_{F}^{*} satisfies

$$\lambda_{\mathrm{F}}^{*} = \begin{cases} \lambda_{F,min}, & T'\left(\lambda_{\mathrm{F}}\right) < 0, \\ \lambda_{\mathrm{F}}^{**}, & T'\left(\lambda_{\mathrm{F}}\right) \text{ has zero point}, \\ \lambda_{F,max}, & T'\left(\lambda_{\mathrm{F}}\right) > 0, \end{cases} \tag{31}$$

$$\lambda_{\mathrm{F}}^{**} = \arg\max \quad T\left(x\right), \quad x \in \left\{\frac{1}{X_3}, \lambda_{\mathrm{F}}^{***}, \frac{1}{Y_3}\right\}, \tag{32}$$

where $\lambda_{\mathrm{F}} \in [\lambda_{F,min}, \lambda_{F,max}]$, $T'\left(\lambda_{\mathrm{F}}\right)$ means the first derivative of $T\left(\lambda_{\mathrm{F}}\right)$ with respect to λ_{F}, $\lambda_{\mathrm{F}}^{***}$ is the solution $T'\left(\lambda_{\mathrm{F}}\right) = 0$, when $T'\left(\lambda_{\mathrm{F}}\right)$ has zero point on $(\lambda_{F,min}, \lambda_{F,max})$.

Proof: From (27) and (28), we know that the first derivative of $T\left(\lambda_{\mathrm{F}}\right)$ with respect to λ_{F} is

$$T'\left(\lambda_{\mathrm{F}}\right) = X_1 e^{-X_2-\lambda_{\mathrm{F}}X_3}\left(1-\lambda_{\mathrm{F}}X_3\right) + Y_1 e^{-Y_2-\lambda_{\mathrm{F}}Y_3}\left(1-\lambda_{\mathrm{F}}Y_3\right). \tag{33}$$

If $T'\left(\lambda_{\mathrm{F}}\right) < 0$, then according to **Theorem 1**, we can know that $T\left(\lambda_{\mathrm{F}}\right)$ can get the maximum value as $\lambda_{\mathrm{F}} = \lambda_{F,min}$, because $T\left(\lambda_{\mathrm{F}}\right)$ is monotonically decrease on $(\lambda_{F,min}, \lambda_{F,max})$, then if $T'\left(\lambda_{\mathrm{F}}\right) > 0$, $T\left(\lambda_{F,max}\right)$ is the maximum value, because $T\left(\lambda_{\mathrm{F}}\right)$ is monotonically increase on $(\lambda_{F,min}, \lambda_{F,max})$. Whereas, we can observe that if $T\left(\lambda_{\mathrm{F}}\right)$ is a continuous bounded function on close set $[\lambda_{F,min}, \lambda_{F,max}]$, and $\exists\lambda_{\mathrm{F}}^{***} \in (\lambda_{F,min}, \lambda_{F,max})$ which leads to $T'\left(\lambda_{\mathrm{F}}^{***}\right) = 0$, $T\left(\lambda_{\mathrm{F}}^{***}\right)$, must be the local maximum or minimum value on $[\lambda_{F,min}, \lambda_{F,max}]$. Then, $T\left(\lambda_{\mathrm{F}}^{**}\right)$ is the maximum value when $\lambda_{\mathrm{F}}^{**} = \arg\max \quad T\left(x\right)$, $x \in \left\{\frac{1}{X_3}, \lambda_{\mathrm{F}}^{***}, \frac{1}{Y_3}\right\}$. So we get λ_{F}^{*}. Considering the constraints of power and the outage probabilities of both macrocell and femtocell transmissions, the optimal FBS density $\lambda_{\mathrm{F}}^{opt}$ for maximizing network throughput is shown in (30). ■

V. Simulation Results

In this section, we evaluate our results for the network throughput of femtocells by simulations. The main simulation parameters are provided in Table I.

TABLE I
Simulation Parameters

Parameters	Abbreviations	Value
α	Path loss	4
P_{Fu}	The power of FUE	25 dBm
P_{Fd}	The power of FBS	30 dBm
$\mathrm{Pr_u}/\mathrm{Pr_d}$	The probability of uplink/downlink communication in a femtocell	0.6/0.4
D_{M0}	Distance from MBS to the typical MUE	100 m
D_{Fu0}	Distance from FUE to the typical FBS	50 m
D_{Fd0}	Distance from FBS to the typical FUE	60 m

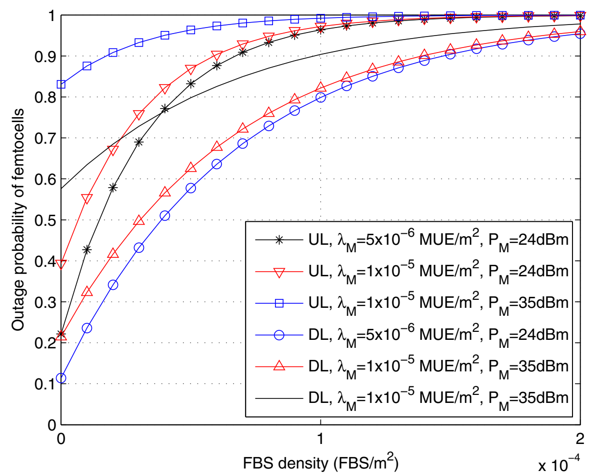

Fig. 2. Outage probability in femtocells

Fig. 2 shows the outage probability of femtocells with different densities of FBS. Initially, we can find that outage probabilities of both uplink and downlink transmissions are increasing as FBS density increases. This is due to the fact that, high FBS density causes more interference to the overall network. Secondly, we observe that, with higher MUE density, the outage probabilities for femtocells is high. The reason is, because high MUE density can generate high interference to the femtocells, leading to more strict constraints in the transmission of femtocells. Similarly, the outage probability is increased when we enlarge the transmission power of MUE, which can cause more interference to the femtocells. Moreover, the uplink transmission power is smaller than the downlink transmission power in femtocells, showing that the FUEs can suffer more interference as receivers in downlink transmission. Therefor, the outage probability of downlink transmission is smaller than that of uplink transmission. In addition, the probabilities of uplink and downlink transmissions in femtocells are set as 0.6 and 0.4, respectively, the uplink transmission in femtocells causes more interference in a HetNets as the FBS density increases. Thus, the outage probabilities of uplink transmission rise faster as compare to that of downlink transmission.

In Fig. 3, we illustrate the relationship between the network throughput of femtocells and FBS density. We observe that, network throughput of femtocell is decreasing as FBS density increases. This increment in FBS density causes more inter-

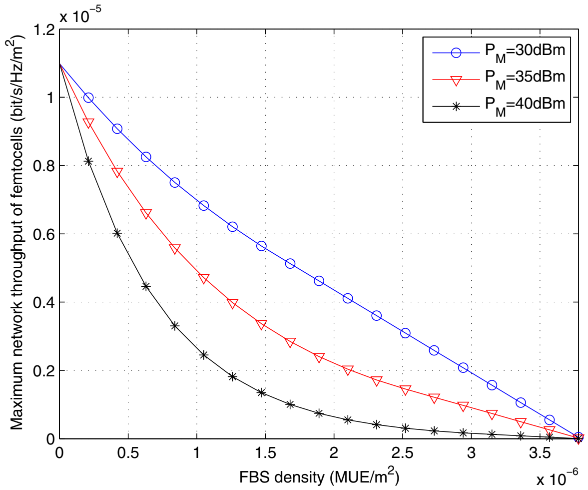

Fig. 3. The maximum network throughput of femtocells vs. Macro cell UE density

ference between femtocells, and the constraints to femtocells become more strict. When FBS density is high enough, the receivers in femtocells severely suffer interference from neighboring receivers, causing network throughput values to reach ”0”. Fig.3 further shows that, high MUE power also cause more interference to femtocells. The maximum network throughput of femtocells is lower while the MUE power is high.In addition, increment in FBSs density, the constraints to femtocells become strict, which makes the optimal FBS density to get only the boundary value of feasible region. Then, the maximum network throughput of femtocells decreases linearly.

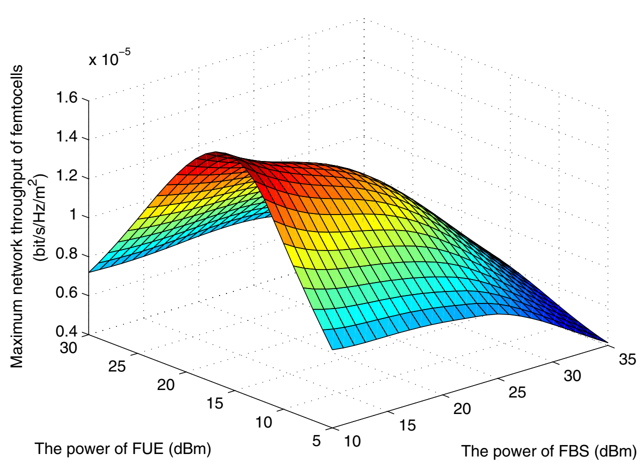

Fig. 4. Maximum network throughput of femtocell vs. femtocell uplink and downlink power

Fig. 4 demonstrates the relationship between the maximum network throughput of femtocells, their uplink, and downlink powers. Clearly, the performance gain for uplink transmission is larger as compare to downlink transmission in femtocells. The reasons are 1) The uplink transmission distance is shorter than the downlink transmission distance, which makes the signal experienced less propagation loss during uplink transmission, 2) Since we set the probabilities of uplink and downlink transmissions as 0.6 and 0.4, respectively, it allows uplink transmissions to achieve high performance gain. It can be observe that, uplink transmission takes more proportion in the overall performance gain. An extreme example in the above figure is, when uplink and downlink transmission are 5dBm and 35dBm, the maximum network throughput of femtocells has a very low value, because the uplink transmission cannot bring much performance gain to the femtocells, and the downlink transmission generates serious interference to the entire network.

VI. Conclusions

In this paper, we optimize the FBS density in HetNets for maximizing network throughput of femtocells. By modeling the HetNets as homogeneous PPPs, we investigate the outage probabilities for macrocell network and femtocells. The network throughput expression of femtocells is evaluated and optimization problem of FBS density is then formulated with the outage probability constraints. We also prove that, the maximum network throughput for femtocells locates in a fixed interval of FBS density. Finally, the optimal FBS density is derived in closed form for maximizing the network throughput for the femtocells in HetNets. The impact of parameters such as the optimal FBS density, MUEs power and the probability of uplink/downlink transmissions in femtocells are discussed through simulation results.

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