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Priori-Aided Channel Tracking for Millimeter-Wave Beamspace Massive MIMO Systems

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***Abstract*—Beamspace MIMO can utilize beam selection to reduce the number of required RF chains in mmWave massive MIMO systems. However, beam selection requires the information of beamspace channel of large size. This is a challenging task, since the user mobility usually leads to the fast variation of mmWave beamspace channels, and the real-time channel estimation involves unaffordable pilot overhead. To solve this problem, in this paper we propose a priori-aided (PA) channel tracking scheme. Specifically, by considering a practical user motion model, we first excavate a temporal variation law of the physical direction between the base station and each mobile user. Then, based on the special sparse structure of mmWave beamspace channels, we propose to utilize the obtained beamspace channels in the previous time slots to predict the prior information of the beamspace channel in the following time slot. Finally, aided by the obtained prior information, the time-varying beamspace channels can be tracked with low pilot overhead. Simulation results verify that the proposed PA channel tracking scheme can achieve much better performance than conventional schemes.**

I. Introduction

The integration of millimeter-wave (mmWave) and massive multiple-input multiple-output (MIMO) has been considered as a key technique for future 5G wireless communications [1], as it can achieve significant increase in data rates due to its wider bandwidth and higher spectral efficiency. Unfortunately, realizing mmWave massive MIMO is not a trivial task. One of the most challenging problems is that each antenna in MIMO systems usually requires one dedicated radio-frequency (RF) chain [1]. As the number of antennas becomes huge and the energy consumption of RF chain is high, this will result in unaffordable hardware cost and energy consumption in mmWave massive MIMO systems [2]. To reduce the number of required RF chains, the concept of beamspace MIMO has been recently proposed [3]. By employing the discrete lens array (DLA), beamspace MIMO can transform the conventional spatial channel into beamspace channel by concentrating the signals from different paths (beams) on different antennas. Since mmWave signals are quasi-optical, the number of effective prorogation paths in mmWave communications is quite limited, which means that the mmWave beamspace channel is sparse [3]. Therefore, we can only select a small number of dominant beams according to the sparse beamspace channel to significantly reduce the effective dimension of MIMO system and thus the number of required RF chains without obvious performance loss [4].

Nevertheless, beam selection requires the base station (BS) to obtain the information of beamspace channel of large size. This is a challenging task, since the user mobility usually leads to the fast variation of mmWave beamspace channels. If we adopt the conventional real-time channel estimation, the pilot overhead will be unaffordable [5]. Therefore, more efficient channel tracking schemes exploiting the temporal correlation of time-varying channels is preferred. The existing channel tracking schemes aim to model the time-varying channels in adjacent time slots by the one-order Markov process, and then the classical Kalman filter can be utilized to track the time-varying channels with low pilot overhead [5]. However, these schemes cannot be directly extended to mmWave beamspace massive MIMO systems, since the beamspace channels with special sparse structure cannot not be modeled by the one-order Markov process. To the best of our knowledge, the channel tracking for mmWave beamspace massive MIMO systems has not been addressed in the literature.

In this paper, we propose a priori-aided (PA) channel tracking scheme for mmWave beamspace massive MIMO systems. Specifically, by considering a practical user motion model [6], we first excavate a temporal variation law of the physical direction between the BS and each mobile user. After that, by combing the proposed temporal variation law and the special sparse structure of mmWave beamspace channels, we propose to utilize the obtained beamspace channels in the previous time slots to predict the prior information, i.e., the support (the index set of non-zero elements in a sparse vector), of the beamspace channel in the following time slot without channel estimation. Finally, with the known supports, the time-varying beamspace channels can be tracked with low pilot overhead. Simulation results verify that the proposed PA channel tracking scheme can achieve much better performance than conventional schemes.

Notation: Lower-case and upper-case boldface letters $\mathbf{a}$ and $\mathbf{A}$ denote a vector and a matrix, respectively; $\mathbf{A}^H$, $\mathbf{A}^{-1}$, and $\mathrm{tr}(\mathbf{A})$ denote the conjugate transpose, inversion, and trace of matrix $\mathbf{A}$, respectively; $|a|$ denotes the amplitude of scalar a; Finally, $\mathrm{Card}\,(\mathcal{A})$ denotes the cardinality of set $\mathcal{A}$.

II. System Model

We consider a typical mmWave massive MIMO system, where the BS employs N antennas and N_{RF} RF chains to simultaneously serve K single-antenna users [4].

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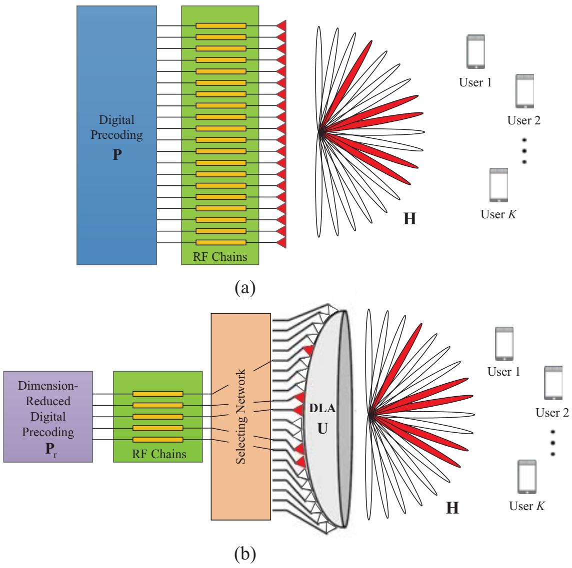

Fig. 1. Comparison of MIMO system architectures: (a) traditional MIMO in the spatial domain; (b) beamspace MIMO.

A. Traditional MIMO in the spatial domain

As shown in Fig. 1 (a), for traditional MIMO in the spatial domain, the $K \times 1$ received signal vector $\mathbf{y}$ for all K users in the downlink can be presented by

$$\mathbf{y} = \mathbf{H}^H \mathbf{P}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_K]$ is the channel matrix, $\mathbf{h}_k$ of size $N \times 1$ is the channel vector between the BS and the kth user as will be discussed in details later, $\mathbf{s}$ of size $K \times 1$ is the signal vector for all K users with normalized power $\mathbb{E}\left(\mathbf{s}\mathbf{s}^H\right) = \mathbf{I}_K$, $\mathbf{P}$ of size $N \times K$ is the precoding matrix with the transmit power constraint $\mathrm{tr}\left(\mathbf{P}\mathbf{P}^H\right) \leq \rho$, $\mathbf{n} \sim \mathcal{CN}\left(0, \sigma^2 \mathbf{I}_K\right)$ is the $K \times 1$ additive white Gaussian noise vector.

Next, we will introduce the channel vector $\mathbf{h}_k$ of the kth user. In this paper, we adopt the widely used Saleh-Valenzuela channel model for mmWave communications as [2]–[4]

$$\mathbf{h}_k = \beta_k^{(0)} \mathbf{a}\left(\psi_k^{(0)}\right) + \sum_{i=1}^{L} \beta_k^{(i)} \mathbf{a}\left(\psi_k^{(i)}\right), \tag{2}$$

where $\beta_k^{(0)} \mathbf{a}\left(\psi_k^{(0)}\right)$ is the line-of-sight (LoS) component of the kth user with $\beta_k^{(0)}$ presenting the complex gain and $\psi_k^{(0)}$ denoting the spatial direction, $\beta_k^{(i)} \mathbf{a}\left(\psi_k^{(i)}\right)$ for $1 \leq i \leq L$ is the ith non-line-of-sight (NLoS) component of the kth user, and L is the total number of NLoS components, $\mathbf{a}(\psi)$ is the $N \times 1$ array steering vector. For the typical uniform linear array (ULA) with N antennas, we have $\mathbf{a}(\psi) = \frac{1}{\sqrt{N}}\left[e^{-j2\pi\psi m}\right]_{m \in \mathcal{I}(N)}$, where $\mathcal{I}(N) = \{l - (N-1)/2,\ l = 0, 1, \cdots, N-1\}$. The spatial direction is defined as $\psi \triangleq \frac{d}{\lambda} \sin\theta$ [3], where θ is the physical direction, λ is the signal wavelength, and d is the antenna spacing satisfying $d = \lambda/2$. Note that the scattering at mmWave frequencies induces more than 20 dB attenuation [1]. Thus, we mainly consider the channel with the LoS component as $\mathbf{h}_k = \beta_k \mathbf{a}(\psi_k)$, where the subscript (0) is omitted.

B. Beamspace MIMO

The conventional spatial channel (2) can be transformed to the beamspace channel by a carefully designed DLA [3] in beamspace MIMO systems as shown in Fig. 1 (b). Essentially, such DLA plays the role of an $N \times N$ spatial discrete fourier transform matrix $\mathbf{U}$, which contains the array steering vectors of N orthogonal directions covering the entire space as $\mathbf{U} = \left[\mathbf{a}\left(\bar{\psi}_1\right), \mathbf{a}\left(\bar{\psi}_2\right), \cdots, \mathbf{a}\left(\bar{\psi}_N\right)\right]^H$, where $\bar{\psi}_n = \frac{1}{N}\left(n - \frac{N+1}{2}\right)$ for $n = 1, 2, \cdots, N$ are the spatial directions pre-defined by DLA. Then, according to Fig. 1 (b), the system model of mmWave beamspace massive MIMO is

$$\tilde{\mathbf{y}} = \mathbf{H}^H \mathbf{U}^H \mathbf{P}\mathbf{s} + \mathbf{n} = \tilde{\mathbf{H}}^H \mathbf{P}\mathbf{s} + \mathbf{n}, \tag{3}$$

where $\tilde{\mathbf{y}}$ is the received signal vector in the beamspace, and the beamspace channel $\tilde{\mathbf{H}}$ is defined as

$$\tilde{\mathbf{H}} = \left[\tilde{\mathbf{h}}_1, \tilde{\mathbf{h}}_2, \cdots, \tilde{\mathbf{h}}_K\right] = \mathbf{U}\mathbf{H} = \left[\mathbf{U}\mathbf{h}_1, \mathbf{U}\mathbf{h}_2, \cdots, \mathbf{U}\mathbf{h}_K\right], \tag{4}$$

where $\tilde{\mathbf{h}}_k$ is the beamspace channel of the kth user. Since almost only LoS component can be used for reliable high-rate transmission in mmWave communications [1], the beamspace channel $\tilde{\mathbf{H}}$ ($\tilde{\mathbf{h}}_k$) at mmWave frequencies enjoys a sparse structure [3]. Therefore, we can select only a small number of dominant beams to reduce the effective dimension of MIMO system without obvious performance loss as $\tilde{\mathbf{y}} \approx \tilde{\mathbf{H}}_{\mathrm{r}}^H \mathbf{P}_{\mathrm{r}} \mathbf{s} + \mathbf{n}$, where $\tilde{\mathbf{H}}_{\mathrm{r}} = \tilde{\mathbf{H}}(l,:)_{l \in \mathcal{B}}$, $\mathcal{B}$ contains the indices of selected beams, $\mathbf{P}_{\mathrm{r}}$ is the dimension-reduced digital precoding matrix. As the dimension of $\mathbf{P}_{\mathrm{r}}$ is much smaller than that of $\mathbf{P}$ in (1), beamspace MIMO can significantly reduce the number of required RF chains as shown in Fig. 1 (b).

III. Beamspace Channel Tracking

In this section, we first excavate a temporal variation law of the physical direction of each mobile user. Then, we propose to use the physical direction to obtain the support of sparse beamspace channel without channel estimation. Finally, a PA channel tracking scheme is proposed.

A. Temporal variation law of the physical direction

Our target is to track the beamspace channel $\tilde{\mathbf{h}}_k$ of user k. As shown in (2)-(4), for $\tilde{\mathbf{h}}_k$, the physical direction θ_k (or the spatial direction ψ_k) of LoS component is a crucial parameter. If we can exploit the temporal variation law of the physical direction, $\tilde{\mathbf{h}}_k$ can be tracked with low pilot overhead

To this end, we consider a linear user motion model[1], and the geometrical relationship between the DLA and the kth mobile user can be presented by Fig. 2, where $r_k(t)$ and $\theta_k(t)$ are the distance and physical direction between the DLA and the kth user at the tth time slot, respectively; T is the time slot interval; v_k and φ_k are the speed and motion direction of the kth user, respectively, which are unknown but not time-varying as we consider that each user moves linearly and uniformly in the linear user motion model. We define a motion state vector $\mathbf{m}_k(t) \triangleq \left[\theta_k(t), \lambda_k(t), \varphi_k\right]^T$ to describe the motion feature

[1]In Section III-C, we will show that the proposed PA channel tracking scheme can be also employed for nonlinear user motion model.

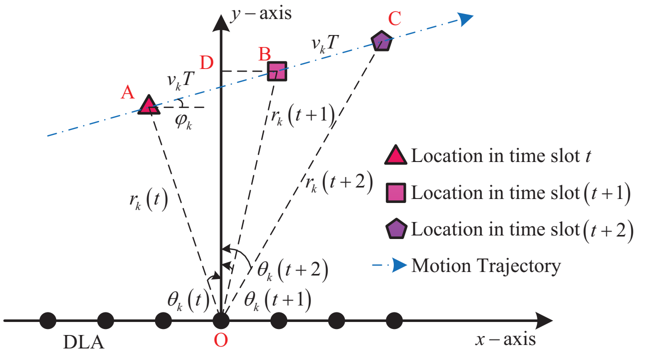

Fig. 2. Geometrical relationship between the DLA and the kth user.

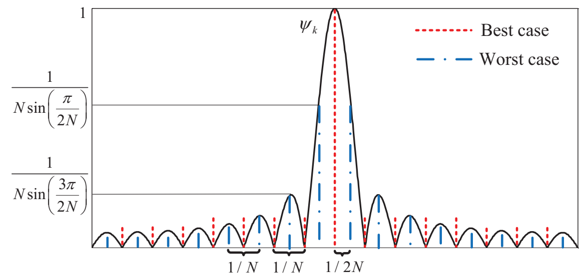

Fig. 3. The normalized amplitude distribution of the elements in $\tilde{\mathbf{h}}_k$.

of user k in time slot t, where $\lambda_k(t) \triangleq \frac{v_k}{r_k(t)}$ can be regarded as the angular speed. Then, we have the following **Lemma 1**.

Lemma 1. *The relationship between* $\mathbf{m}_k(t)$ *and* $\mathbf{m}_k(t+1)$ *can be presented as*

$$\begin{aligned}\mathbf{m}_k(t+1) &= \Theta(\mathbf{m}_k(t)) \qquad (5)\\ &= \begin{bmatrix} \arctan\left\{\frac{\sin[\theta_k(t)]+T\lambda_k(t)\cos\varphi_k}{\cos[\theta_k(t)]+T\lambda_k(t)\sin\varphi_k}\right\} \\ \frac{\lambda_k(t)}{\sqrt{1+2T\lambda_k(t)\sin[\theta_k(t)+\varphi_k]+T^2\lambda_k^2(t)}} \\ \varphi_k \end{bmatrix},\end{aligned}$$

where $\Theta(\mathbf{m}_k(t))$ *is a function of* $\mathbf{m}_k(t)$.

Proof: See [6]. ■

Our next task is to reformulate $\lambda_k(t)$ and φ_k in $\mathbf{m}_k(t)$ by the physical direction. To this end, we can observe the triangles OAB and OAC in Fig. 2. Then, by using the law of sine, we have the following two groups of equations as

$$\begin{cases} \lambda_k(t+1) = \frac{v_k}{d_k(t+1)} = \frac{\sin[\theta_k(t+1)-\theta_k(t)]}{T\cos[\theta_k(t)+\varphi_k]} \\ \lambda_k(t) = \frac{v_k}{d_k(t)} = \frac{\sin[\theta_k(t+1)-\theta_k(t)]}{T\cos[\theta_k(t+1)+\varphi_k]} \end{cases} \qquad (6)$$

$$\begin{cases} \lambda_k(t+2) = \frac{v_k}{r_k(t+2)} = \frac{\sin[\theta_k(t+2)-\theta_k(t)]}{2T\cos[\theta_k(t)+\varphi_k]} \\ \lambda_k(t) = \frac{v_k}{r_k(t)} = \frac{\sin[\theta_k(t+2)-\theta_k(t)]}{2T\cos[\theta_k(t+2)+\varphi_k]} \end{cases} \qquad (7)$$

According to (6) and (7), we have

$$\lambda_k(t+2) = \frac{\sin[\theta_k(t+2)-\theta_k(t)]}{2T\cos[\theta_k(t)+\varphi_k]}, \qquad (8)$$

$$\varphi_k = \frac{2a_k\cos[\theta_k(t+2)] - b_k\cos[\theta_k(t+1)]}{2a_k\sin[\theta_k(t+2)] - b_k\sin[\theta_k(t+1)]}, \qquad (9)$$

where we define $a_k \triangleq \sin[\theta_k(t+1)-\theta_k(t)]$ and $b_k \triangleq \sin[\theta_k(t+2)-\theta_k(t)]$. Based on (8), (9), and **Lemma 1**, we can conclude that once we have estimated the physical directions in time slots t, $(t+1)$, and $(t+2)$, the physical direction in the following time slot $(t+3)$ can be predicted in advance without channel estimation.

B. The support of beamspace channel

In this subsection, we propose to use the physical direction to obtain the support of beamspace channel without channel estimation. This is achieved by exploiting the special sparse structure of mmWave beamspace channels, which is proved by the following **Lemma 2**.

Lemma 2. *Consider the beamspace channel* $\tilde{\mathbf{h}}_k$ *of user* k*, and assume* V *is an even integer without loss of generality. The ratio between the power* P_V *of* V *strongest elements of* $\tilde{\mathbf{h}}_k$ *and the total channel power* P_T *can be lower-bounded by*

$$\frac{P_V}{P_T} \geq \frac{2}{N^2}\sum_{i=1}^{V/2}\frac{1}{\sin^2\left(\frac{(2i-1)\pi}{2N}\right)}. \qquad (10)$$

Moreover, once the position n_k^* *of the strongest element of* $\tilde{\mathbf{h}}_k$ *is determined, the other* $V-1$ *strongest elements will uniformly locate around it with the interval* $1/N$.

Proof: Based on (2)-(4), the beamspace channel $\tilde{\mathbf{h}}_k$ can be presented as $\tilde{\mathbf{h}}_k = \beta_k\left[\Upsilon(\bar{\psi}_1-\psi_k),\cdots,\Upsilon(\bar{\psi}_N-\psi_k)\right]^H$, where $\Upsilon(x) \triangleq \frac{\sin N\pi x}{N\sin\pi x}$. Fig. 3 shows the normalized amplitude (without β_k) distribution of the elements in $\tilde{\mathbf{h}}_k$, where the set of red dash lines (or blue dot dash lines) presents the spatial directions $\bar{\psi}_n = \frac{1}{N}\left(n-\frac{N+1}{2}\right)$ for $n=1,2,\cdots,N$ pre-defined by DLA. From Fig. 3, we can observe that when the practical spatial direction ψ_k exactly equals to one pre-defined spatial direction, there is only one strongest element containing all the power of $\tilde{\mathbf{h}}_k$, which is the best case. In contrast, the worst case will happen when the distance between ψ_k and one pre-defined spatial direction is equal to $1/2N$. In this case, the power P_V of V strongest elements of $\tilde{\mathbf{h}}_k$ is

$$P_V = \frac{2\beta_k^2}{N^2}\sum_{i=1}^{V/2}\frac{1}{\sin^2\left(\frac{(2i-1)\pi}{2N}\right)}. \qquad (11)$$

According to (11) and the fact that the total channel power P_T is normalized to β_k^2, we can conclude that P_V/P_T is lower-bounded by (10). Moreover, as we can also observe from Fig. 3, once the position n_k^* of the strongest element of $\tilde{\mathbf{h}}_k$ is determined, the other $V-1$ strongest elements will uniformly locate around it with the interval $1/N$. ■

From **Lemma 2**, we can derive two conclusions. The first one is that $\tilde{\mathbf{h}}_k$ can be considered as a sparse vector, since the most power of $\tilde{\mathbf{h}}_k$ is focused on a small number of dominant elements. For example, when $N=256$ and $V=16$, the lower-bound of P_V/P_T is about 98%. The second one is that the support of $\tilde{\mathbf{h}}_k$ can be uniquely determined by n_k^* as

$$\mathrm{supp}\left(\tilde{\mathbf{h}}_k\right) = \mathrm{mod}_N\left\{n_k^*-\frac{V}{2},\cdots,n_k^*+\frac{V-2}{2}\right\}, \qquad (12)$$

where $\mathrm{Card}\left(\mathrm{supp}\left(\tilde{\mathbf{h}}_k\right)\right) = V$, and $\mathrm{mod}_N(\cdot)$ is the modulo operation with respect to N, which guarantees that all indices in $\mathrm{supp}\left(\tilde{\mathbf{h}}_k\right)$ belong to $\{1,2,\cdots,N\}$.

Conventional channel estimation $1 \leq t \leq 3$
1) Estimate the beamspace channel $\tilde{\mathbf{h}}_k(t)$;
2) Approximate the physical direction $\theta_k(t)$ as
$\theta_k(t) \approx \arcsin\frac{\lambda}{Nd}\left(n_k^*(t) - \frac{(N+1)}{2}\right)$;
Channel tracking $t > 3$
3) Predict $\theta_k(t)$ using $\theta_k(t-3)$, $\theta_k(t-2)$, and $\theta_k(t-1)$;
4) Detect $\text{supp}\left(\tilde{\mathbf{h}}_k(t)\right)$ according to (12) and (13);
5) Estimate the nonzero elements of $\tilde{\mathbf{h}}_k(t)$;
6) Refine the physical direction $\theta_k(t)$ based on $n_k^*(t)$;

Algorithm 1: The proposed PA channel tracking

According to the proof of **Lemma 2**, we know that n_k^* depends on ψ_k (or θ_k), where the relationship is

$$n_k^* = \arg\min_{1\leq n\leq N} |\bar{\psi}_n - \psi_k| = \arg\min_{1\leq n\leq N} \left|\bar{\psi}_n - \frac{d}{\lambda}\sin\theta_k\right|. \quad (13)$$

Therefore, once we have obtain θ_k, $\text{supp}\left(\tilde{\mathbf{h}}_k\right)$ can be directly obtained based on (12) and (13) without channel estimation.

C. PA channel tracking scheme

Based on the conclusions derived above, the PA channel tracking scheme can be summarized in **Algorithm 1**, which is divided into two parts. The first part is the conventional channel estimation in the first 3 time slots. During the first part, $\tilde{\mathbf{h}}_k(t)$ is estimated in step 1 using conventional beamspace channel estimation schemes, such as the one proposed in [7]. Then, in step 2, $\psi_k(t)$ is approximated as $\psi_k(t) \approx \bar{\psi}_{n_k^*(t)}$ according (13), and equivalently, $\theta_k(t)$ can be computed by

$$\theta_k(t) \approx \arcsin\frac{\lambda}{d}\bar{\psi}_{n_k^*(t)} = \arcsin\frac{\lambda}{Nd}\left(n_k^*(t) - \frac{(N+1)}{2}\right). \quad (14)$$

After the physical directions in the first 3 time slots have been obtained, the channel tracking in the second part can be executed. Specifically, in the step 3, we first utilize $\theta_k(t-3)$, $\theta_k(t-2)$, and $\theta_k(t-1)$ to predict $\theta_k(t)$ based on (8), (9), and **Lemma 1**. After that, in step 4, we detect $\text{supp}\left(\tilde{\mathbf{h}}_k(t)\right)$ according to (12) and (13) without channel estimation. Then, the nonzero elements of $\tilde{\mathbf{h}}_k(t)$ are estimated by least squares algorithm in step 5, which only requires low pilot overhead since the number of nonzero elements V is much smaller than N. Finally, after $\tilde{\mathbf{h}}_k(t)$ has been tracked, $\theta_k(t)$ is refined by $n_k^*(t)$ based on (14) for two reasons: i) the impact of error propagation induced by the approximation in step 2 can be avoid; ii) the proposed PA channel tracking scheme can be employed in nonlinear user motion model, since the deviation induced by the prediction in step 3 can be adaptively modified when the motion direction has changed.

IV. Simulation Results

In this section, we consider a typical mmWave beamspace massive MIMO system, where the BS equips an $N = 256$-element DLA with $d = \lambda/2$ and $N_{\text{RF}} = 4$ RF chains to simultaneously serve $K = 4$ users. For each user in each time slot, we regard the beamspace channel as a sparse vector with sparsity $V = 16$, and assume that the complex channel gain follows $\mathcal{CN}(0,1)$. We totally observe 30 time slots with the time slot interval $T = 1$. The motion states of users 1-3 in the initial time slot 1 are set as $\mathbf{m}_1(1) = \left[\frac{\pi}{9}, 0.0154, \frac{3\pi}{4}\right]^T$, $\mathbf{m}_2(1) = \left[-\frac{\pi}{9}, 0.0071, \frac{\pi}{6}\right]^T$, and $\mathbf{m}_3(1) = \left[-\frac{2\pi}{9}, 0.0114, 0\right]^T$, respectively. For user 4, we consider a nonlinear motion model, with $\mathbf{m}_4(1) = \left[0, 0.074, \frac{-3\pi}{4}\right]^T$ in time slots 1-15 and $\mathbf{m}_4(15) = \left[\theta_1(15), \lambda_1(15), 0\right]^T$ in time slots 15-30.

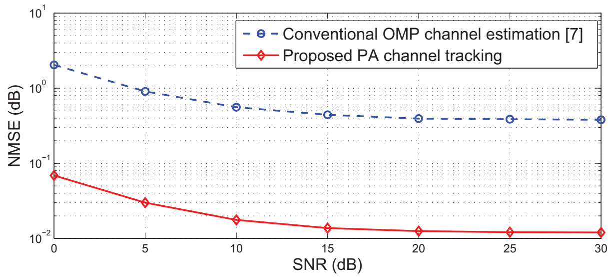

Fig. 4. NMSE performance comparison against SNR.

Fig. 4 compares the normalized mean square error (NMSE) performance between the proposed PA channel tracking scheme and the conventional real-time orthogonal matching pursuit (OMP) channel estimation scheme [7]. For fair comparison, we set $Q = 16$ pilots per time slot for both the two schemes. From Fig. 4, we can observe that the proposed PA channel tracking scheme can achieve much better NMSE performance than conventional OMP channel estimation scheme. In other words, to achieve the same NMSE performance, PA channel tracking scheme requires much lower pilot overhead and SNR than conventional OMP channel estimation scheme.

V. Conclusions

In this paper, by utilizing the temporal variation law of the physical direction and the special sparse structure of mmWave beamspace channels, we propose a PA channel tracking scheme for mmWave beamspace massive MIMO systems. Simulation results verify that to achieve the same NMSE performance, the proposed PA channel tracking scheme requires much lower pilot overhead and SNR than conventional OMP channel estimation scheme.

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