On the Spectral Efficiency of Space-Constrained Massive MIMO with Linear Receivers

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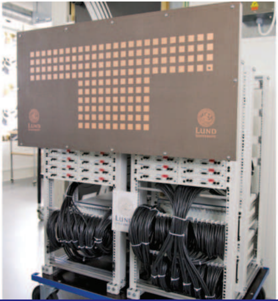
Outline

Introduction I

- **Favorable propagation:** the channel vectors between the different UEs and the BS become asymptotically orthogonal with a large antenna array.
  - The inter-element spacing is more than half a wavelength
  - Suffer from less spatial correlation
- **Space constrained:** the dense deployment of a massive number of antennas in a limited physical space.
  - The inter-element spacing is less than half a wavelength
  - Suffer from increased spatial correlation



System Model I

- We consider the uplink of a single-cell massive MIMO system with M BS antennas and K single-antenna UEs. The received vector $\mathbf{y} \in \mathbb{C}^{M\times 1}$ given by

$$\mathbf{y} = \sqrt{p_u}\mathbf{G}\mathbf{x} + \mathbf{n}, \tag{1}$$

  where p_u is the average power of each UE, $\mathbf{x} \in \mathbb{C}^{K\times 1}$ denotes the zero-mean Gaussian transmit vector from all K UEs with unit average power, and the elements of $\mathbf{n}$ represent the additive white Gaussian noise (AWGN) with zero-mean and unit variance.

System Model II

- The channel matrix between the BS and UEs can be written as $\mathbf{G} = \mathbf{AHD}^{1/2}$, where $\mathbf{H} \in \mathbb{C}^{P \times K}$ is the propagation response matrix standing for small-scale fading, and $\mathbf{D} \in \mathbb{C}^{K \times K}$ denotes a diagonal matrix whose kth diagonal element ζ_k models the large-scale fading (including geometric attenuation and shadow fading) of the kth UE.
- We assume that large-scale fading ζ_k are constant. Moreover, $\mathbf{A} \in \mathbb{C}^{M \times P}$ is the transmit steering matrix, with P denoting a large but finite number of incident directions in the propagation channel.
- For the sake of analytical simplicity, we assume that all UEs are seen from the same set of directions with cardinality P.

System Model II

- Considering the widely used uniform linear antenna array, we can write $\mathbf{A}$ as

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_P)], \tag{2}$$

where $\mathbf{a}(\theta_i)$, for $i = 1, 2, \ldots, P$ denotes a length-M normalized steering vector as

$$\mathbf{a}(\theta_i) = \frac{1}{\sqrt{P}} \left[1, e^{-j\frac{2\pi d}{\lambda}\sin\theta_i}, \ldots, e^{-j\frac{2\pi d}{\lambda}(M-1)\sin\theta_i}\right]^T, \tag{3}$$

where d is the antenna spacing, λ denotes the carrier wavelength, and θ_i represents the direction of arrival (DOA).

- The normalized total antenna array space d_0 at the BS can be expressed as $d_0 = \frac{dM}{\lambda}$. In (3), we use the factor $\frac{1}{\sqrt{P}}$ to normalize the steering vector $\mathbf{a}(\theta_i)$.

System Model III

- The linear receiver matrix $\mathbf{T} \in \mathbb{C}^{M \times K}$ is used to separate the received signal into K streams by

$$\mathbf{r} = \mathbf{T}^H \mathbf{y} = \sqrt{p_u} \mathbf{T}^H \mathbf{G} \mathbf{x} + \mathbf{T}^H \mathbf{n}. \tag{4}$$

- The kth element of the received signal vector is given by

$$r_k = \sqrt{p_u} \mathbf{t}_k^H \mathbf{g}_k x_k + \sqrt{p_u} \sum_{l \neq k}^{K} \mathbf{t}_k^H \mathbf{g}_l x_l + \mathbf{t}_k^H \mathbf{n}. \tag{5}$$

- The achievable uplink SE, R_k, of the kth UE is given by

$$R_k = \mathrm{E}\left\{ \log_2 \left(1 + \frac{p_u |\mathbf{t}_k^H \mathbf{g}_k|^2}{p_u \sum_{l \neq k}^{K} |\mathbf{t}_k^H \mathbf{g}_l|^2 + \|\mathbf{t}_k\|^2} \right) \right\}. \tag{6}$$

- The uplink sum SE can be then defined as

$$R = \sum_{k=1}^{K} R_k \quad \text{in bits/s/Hz}. \tag{7}$$

Proposition 1

For space-constrained massive MIMO systems with MRC receivers, the approximated sum achievable SE is given by

$$R^{MRC} \approx \sum_{k=1}^{K} \log_2 \left(1 + \frac{p_u \left(M^2 + \sum_{i=1}^{P} \beta_i^2 \right) \zeta_k}{p_u \sum_{l \neq k}^{K} \zeta_l \sum_{i=1}^{P} \beta_i^2 + M \zeta_k} \right), \tag{8}$$

where β_i is the ith eigenvalue of the matrix $\mathbf{A}^H \mathbf{A}$.

MRC Receivers II

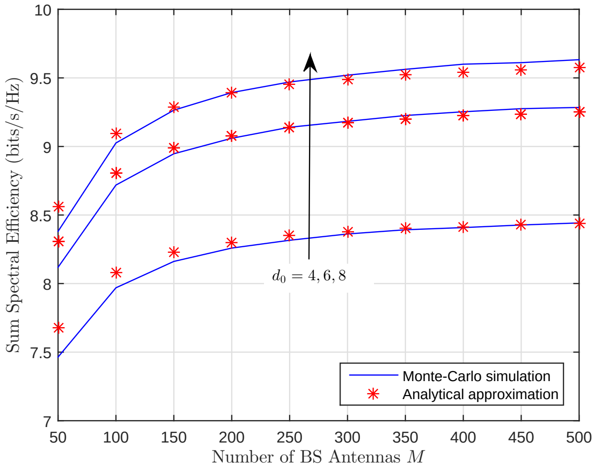
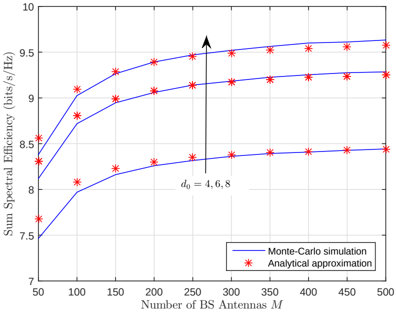


Figure 1: Simulated and analytical approximation of the sum SE of massive MIMO with MRC receivers against the number of BS antennas ($P = 12$ and $K = 6$).

MRC Receivers III



- The sum SE saturates with an increasing number of BS antennas for MRC receivers
- For the same number of BS antennas, a monotonic increase in the sum SE is achieved as d_0 becomes larger
- The gap between the curves decreases as d_0 increases, which implies that the effect of constrained space becomes less pronounced.

ZF Receivers I

Proposition 2

For space-constrained massive MIMO systems with ZF receivers, the achievable sum SE is lower bounded as

$$R_L^{ZF}=\sum_{k=1}^{K}\log_2\left(1+p_u\zeta_k\exp\left(\sum_{n\neq k}^{K}\zeta_n\left(\psi(K)+\frac{|\mathbf{Y}_{P-K+1}|}{\prod_{i<j}^{P}\left(\beta_j-\beta_i\right)}\right)-\left(\psi(n)+\frac{\sum\limits_{n=P-K+2}^{P}|\mathbf{Y}_n|}{\prod_{i<j}^{P}\left(\beta_j-\beta_i\right)}\right)\right)\right), \tag{9}$$

where $\psi(\cdot)$ is the digamma function, and $\mathbf{Y}_n$ denotes a $P\times P$ matrix whose entries are

$$[\mathbf{Y}_n]_{p,q}=\begin{cases}\beta_p^{q-1}, & q\neq n,\\ \beta_p^{q-1}\ln\beta_p, & q=n.\end{cases} \tag{10}$$

ZF Receivers II

Proposition 3

For space-constrained massive MIMO systems with ZF receivers, the achievable sum SE is upper bounded as

$$K\log_2\left(\frac{|\mathbf{\Delta}_2|}{\prod_{i=1}^{K-1}\Gamma(K-i)\prod_{i<j}^{P}\left(\beta_j-\beta_i\right)}+\frac{p_u|\mathbf{\Delta}_1|}{\prod_{i=1}^{K}\Gamma(K-i+1)\prod_{i<j}^{P}\left(\beta_j-\beta_i\right)}\right)-\frac{K}{\ln 2}\left(\sum_{n=1}^{K-1}\psi(n)+\frac{\sum\limits_{n=P-K+2}^{P}|\mathbf{Y}_n|}{\prod_{i<j}^{P}\left(\beta_j-\beta_i\right)}\right),$$

where $\mathbf{\Delta}_1=[\mathbf{\Xi}_1\mathbf{\Phi}_1]$ *is a* $P\times P$ *matrix with entries*

$$[\mathbf{\Xi}_1]_{p,q}=\beta_p^{q-1},\quad q=1,2,\ldots,P-K,$$

$$[\mathbf{\Phi}_1]_{p,q}=\beta_p^{q}\Gamma(q-P+K+1),\quad q=P-K+1,\ldots,P,$$

and $\mathbf{\Delta}_2=[\mathbf{\Xi}_2\mathbf{\Phi}_2]$ *is a* $P\times P$ *matrix with entries*

$$[\mathbf{\Xi}_2]_{p,q}=\beta_p^{q-1},\quad q=1,2,\ldots,P-K+1,$$

$$[\mathbf{\Phi}_2]_{p,q}=\beta_p^{q}\Gamma(q-P+K),\quad q=P-K+2,\ldots,P.$$

ZF Receivers III

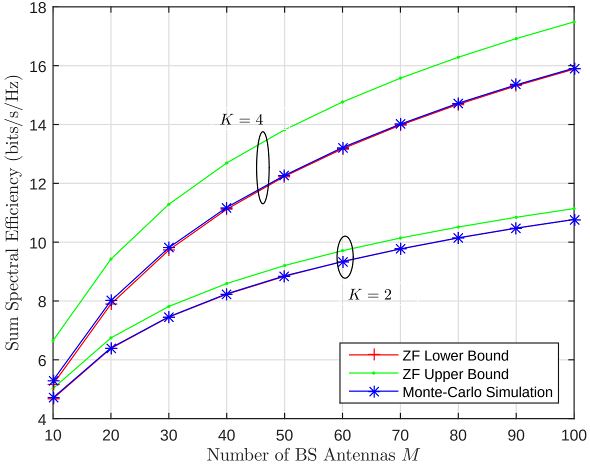
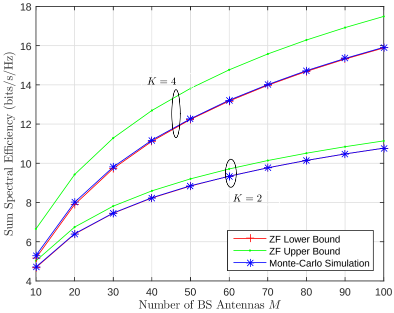


Figure 2: Simulated and analytical approximation of the sum SE of massive MIMO with ZF receivers against the number of BS antennas ($P = 12$ and $d_0 = 4$).

ZF Receivers IV



- All lower bounds can predict the exact sum SE for all the considered cases, which validate their tightness
- The upper bounds are relatively looser, due to the large variance of the involved random variables
- Adding more antennas significantly improves the sum SE by suppressing thermal noise

ZF Receivers V

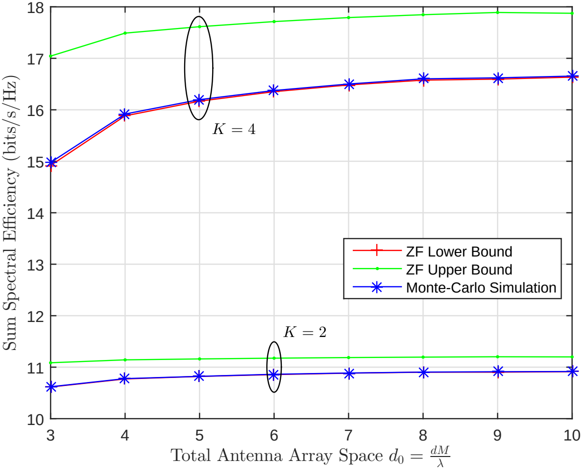
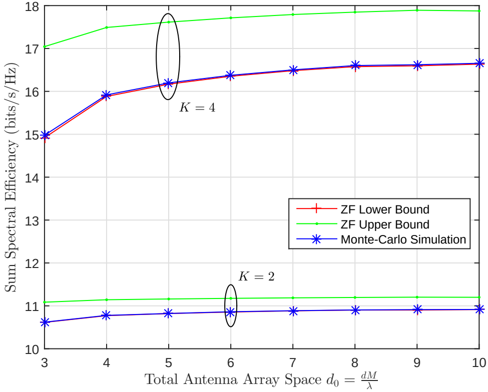


Figure 3: Simulated and analytical approximation of the sum SE of massive MIMO with ZF receivers against the total antenna array space $d_0 = \frac{dM}{\lambda}$ ($M = 100$ and $P = 12$.

ZF Receivers IV



- The sum SE does improve with increased total physical space, particularly for the case of more UEs

MMSE Receivers I

Proposition 4

For space-constrained massive MIMO systems with MMSE receivers, the exact sum SE is given by

$$R^{MMSE} = \frac{K\log_2 e}{\prod_{i<j}^{P}(\beta_j - \beta_i)} \sum_{l=1}^{P} \sum_{n=P-K+1}^{P} \beta_l^{n-1} e^{1/\beta_l p_u} \times D_{l,n} E_{n-P+K}\left(\frac{1}{\beta_l p_u}\right), \quad (11)$$

where $D_{l,n}$ is the (l,n)th cofactor of a $P \times P$ matrix $\mathbf{D}$ with the (p,q)th entry $[\mathbf{D}]_{p,q} = \beta_p^{q-1}$, and $E_x(y)$ is the exponential integral function.

MMSE Receivers II

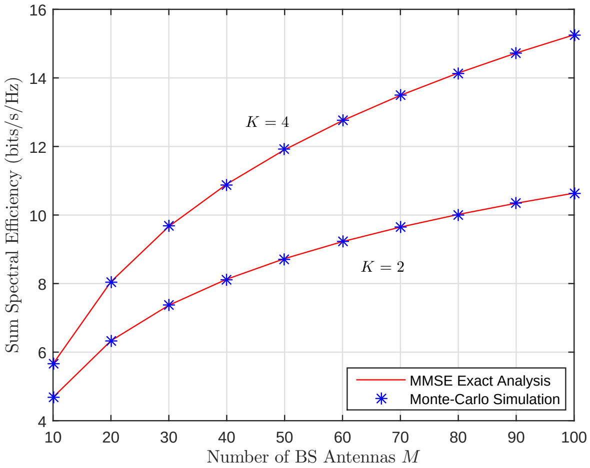
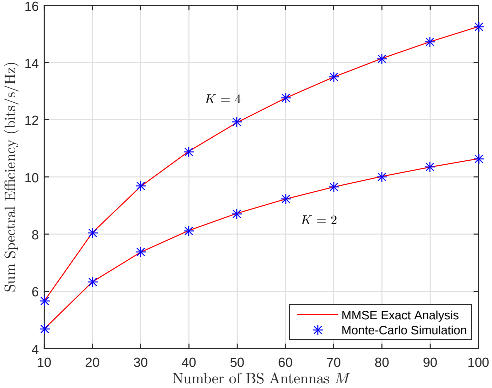


Figure 4: Simulated and analytical expression of the sum SE of massive MIMO with MMSE receivers against the number of antennas at BS ($P = 12$ and $d_0 = 4$).

ZF Receivers III



- The exact analytical results are indistinguishable from the numerical simulations, which validates the correctness of the derived expressions

MMSE Receivers IV

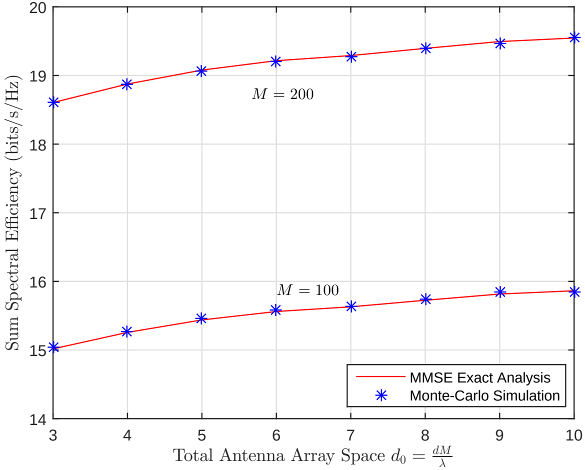
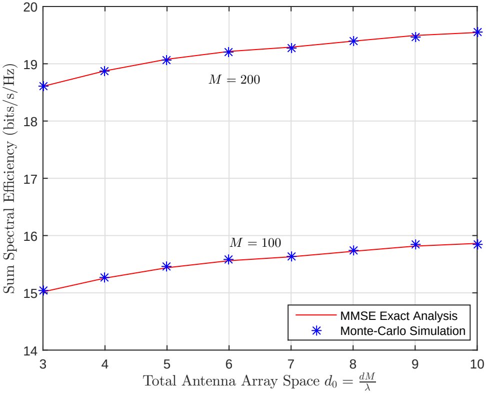


Figure 5: Simulated and analytical expression of the sum SE of massive MIMO with MMSE receivers against the total antenna array space $d_0 = \frac{dM}{\lambda}$ ($K = 4$ and $P = 8$).

ZF Receivers V



- With a fixed total antenna array space, the sum SE can be still increased by employing more BS antennas. This is because the improved array gain caused by the increased M dominates the sum SE loss due to the reduced d_0.

Conclusions

- We investigated the performance of massive MIMO systems with a practical space-constrained topology
- We first derived the approximated sum SE with MRC receivers; A saturation of the achievable sum SE occurs with an increasing number of BS antennas
- For ZF receivers, we derived new lower and upper bounds on the sum SE, which increases for a higher number of UEs, as long as $M \gg K$. Moreover, the proposed lower bound is tighter than the upper bound
- For MMSE receivers, an exact expression for the sum SE is derived and validated by simulation results, which shows that the sum SE increases with the number of BS antennas
- ZF and MMSE receivers work well for space-constrained massive MIMO systems, while MRC receivers can only work well at low SINRs

For Further Reading

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Thank you!