Enhancing Spectrum Efficiency for Near-Field Communications: Applying Near-Field NOMA

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***Abstract*—Extremely large-scale multiple-input multiple-output (XL-MIMO) has been considered as a potential key technology for spectrum efficiency enhancement in 6G communications. The XL-MIMO systems introduce spherical-wave based near-field communications and a new multiple access scheme named location division multiple access (LDMA) is adopted. The near-field LDMA communications provides a novel distance dimension for enhancing spectrum efficiency. However, the spectrum efficiency enhancement in near-field LDMA communications is limited, when user are located far away from the base station and the channels become highly correlated. To solve this problem, we apply non-orthogonal multiple access (NOMA) in near-field communications to further enhance spectrum efficiency. Specifically, for two-user near-field NOMA communications, the closed-form solutions for spectrum efficiency in near-field LDMA and near-field NOMA systems and the impact of channel correlation on them are analyzed. Besides, the criteria for applying near-field NOMA is formulated. Moreover, we extend the two-user near-field NOMA to the multi-user scenarios and give a overall framework of applying NOMA in near-field multi-user communications. Finally, simulation results are provided to verify the feasibility and superiority of applying NOMA in near-field communications to enhance spectrum efficiency.**

***Index Terms*—Near-field communications, location division multiple access (LDMA), non-orthogonal multiple access (NOMA), beamfocusing.**

I. Introduction

To facilitate the emerging applications in the sixth-generation (6G) networks such as extended reality, it is predicted that 6G will achieve 10-fold improvement in spectrum efficiency compared to 5G [1]. Extremely large-scale multiple-input multiple-output (XL-MIMO) has been considered as a potential key technology for spectrum efficiency enhancement in 6G [2]. Different from classic 5G massive MIMO systems, XL-MIMO deploys extremely large-scale antenna arrays (ELAA) with hundreds even thousands of antennas.

The XL-MIMO systems implemented with ELAA not only bring the increasing number of antennas, but also a shift of the propagation model of the wireless communications. Specifically, the substantial rise in both the number of antennas and carrier frequencies in 6G XL-MIMO communications will significantly enlarge the near-field region and more users are likely to be located in near-field region [2]. In the near-field region, the spherical-wave model is used to characterize the electromagnetic wave propagation instead of planar-wave model used in the far-field.

The transformation in propagation models presents the additional focusing ability in the distance domain, offering opportunities for spectrum efficiency enhancement in near-field communications. Specifically, different from the classical far-field beamforming which steers beams towards specific angles like a flashlight, the near-field beamforming can concentrate the beam energy at specific locations like a spotlight, which is also be called *near-field beamfocusing* [3]. Compared to classical spatial division multiple access (SDMA), a multiple access scheme in the near-field region named location division multiple access (LDMA) is proposed in [4]. LDMA utilizes the beamfocusing property of near-field beams to mitigate the inter-user interferences and serve multiple users at identical angle but different distances simultaneously. In other words, LDMA provide a novel distance dimension for improving spectrum efficiency in wireless communication systems.

When the distance between the user and the base station (BS) is sufficiently close, LDMA for users at the same angle but different distances can be achieved with excellent performance. However, for users located relatively far from the BS, the channel correlation is medium and the linear precoding for LDMA presents poor performance [4]. Consequently, how to further enhance spectrum efficiency of these users is a critical problem.

To tackle this problem, in this paper we apply non-orthogonal multiple access (NOMA) in near-field communications to further enhance spectrum efficiency. Specifically, the near-field NOMA systems can possess stronger interference mitigation ability where the successive interference cancellation (SIC) is carried out to avoid the performance degradation in near-field LDMA systems. First, for the simplified two-user models, the closed-form solutions for spectrum efficiency in near-field LDMA and NOMA systems and the impact of channel correlation on them are analyzed. Besides, the criteria for applying near-field NOMA is formulated. Additionally, we extend the two-user near-field NOMA communications to the general multi-user scenarios and give a overall framework of applying NOMA in near-field multi-user communications. Different from the work where the pre-configured beams for near-field users are used to additionally serve far-field users [5], [6], our goal is to improve the overall spectrum efficiency of all users. Simulation results demonstrate the superiority of applying near-field NOMA in terms of spectrum efficiency in near-field communications.

II. System Model

A. System Model

We consider a single-cell downlink XL-MIMO communication system, where the BS equipped with an N-antenna uniform linear array (ULA) and N_{RF} RF chains serves K users with single-antenna.

It is assumed that the K users are divided into N_{RF} groups and the users in the same group are served by the same RF chains or the same beam applying NOMA [7]. Let M_n for $n = 1, 2, \cdots, N_{\mathrm{RF}}$ denote the nth set of user groups and $\mathbf{w}_n \in \mathbb{C}^{N\times 1}$ denotes the effective precoding vector for M_n, where $\mathbf{w}_n = \mathbf{W}_{\mathrm{A}}\mathbf{w}_{\mathrm{D},n}$, $\mathbf{W}_{\mathrm{A}} \in \mathbb{C}^{N\times N_{RF}}$ denotes the analog precoder matrix and $\mathbf{w}_{\mathrm{D},n}$ is the nth column of the digital precoder matrix $\mathbf{W}_{\mathrm{D}} \in \mathbb{C}^{N_{RF}\times N_{RF}}$. Without loss of generality, we assume that we have gotten the perfect channel state information (CSI) and the effective channel gains can be arranged in the descend order, i.e., $|\mathbf{h}_{1,n}^H\mathbf{w}_n|^2 \geq |\mathbf{h}_{2,n}^H\mathbf{w}_n|^2 \geq \cdots \geq |\mathbf{h}_{|M_n|,n}^H\mathbf{w}_n|^2$. Let us suppose that the mth user and the ith user are in the same user group, satisfying $1 \leq m < i \leq |M_n|$. Then the mth user with high effective channel gains can firstly carry out SIC, detecting the signals of the ith user with low effective channel gains and removing them [7]. The received signal at the mth user in the nth beam after performing SIC can be expressed as

$$\begin{aligned} y_{m,n} =& \mathbf{h}_{m,n}^H\mathbf{w}_n\sqrt{p_{m,n}}s_{m,n} + \mathbf{h}_{m,n}^H\mathbf{w}_n\sum_{i=1}^{m-1}\sqrt{p_{i,n}}s_{i,n} \\ &+ \mathbf{h}_{m,n}^H\sum_{j\neq n}\sum_{i=1}^{|M_j|}\mathbf{w}_j\sqrt{p_{i,j}}s_{i,j} + n_{m,n}, \end{aligned} \tag{1}$$

where $s_{m,n}$ and $p_{m,n}$ are the transmitted signal and transmitted power for the mth user in the nth beam, $\mathbf{h}_{m,n}$ is the downlink channel vector, and $n_{m,n}$ is the noise following the distribution $\mathcal{CN}\left(0, \sigma^2\right)$. Therefore, the SINR at the mth user in the nth beam can be expressed as

$$\gamma_{m,n} = \frac{|\mathbf{h}_{m,n}^H\mathbf{w}_n|^2 p_{m,n}}{|\mathbf{h}_{m,n}^H\mathbf{w}_n|^2\sum\limits_{i=1}^{m-1}p_{i,n} + \sum\limits_{j\neq n}|\mathbf{h}_{m,n}^H\mathbf{w}_j|^2\sum\limits_{i=1}^{|M_j|}p_{i,j} + \sigma^2}, \tag{2}$$

According to (2), the achievable rate at the mth user in the nth beam is

$$R_{m,n} = \log_2\left(1 + \gamma_{m,n}\right). \tag{3}$$

B. Far-Field NOMA and Near-Field NOMA

Generally, the channel model can be divided into the far-field and the near-field channel model by the electromagnetic wave propagation characteristics and the Rayleigh distance is usually adopted as the boundary [2]. In 5G massive MIMO systems, the planar-wave based Saleh-Valenzuela channel model is widely adopted and the far-field channel $\mathbf{h}_k^{\mathrm{far}}$ can be expressed as

$$\mathbf{h}_k^{\mathrm{far}} = \sqrt{N}\alpha_0\mathbf{a}(\theta_0) + \sqrt{\frac{N}{L}}\sum_{l=1}^{L}\alpha_l\mathbf{a}(\theta_l), \tag{4}$$

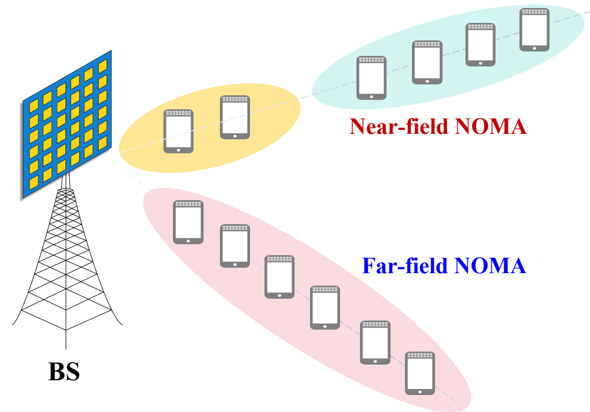

Fig. 1. Comparison between near-field and far-field NOMA systems.

where α_0, θ_0, α_l, θ_l, L denote the complex gain and the angle-of-departure (AoD) of the line-of-sight (LoS) path, the complex gain and the AoD of the non-line-of-sight (NLoS) paths, and the total number of the NLoS paths, respectively.

For the ULA, the beam steering vector $\mathbf{a}(\theta)$ can be expressed as

$$\mathbf{a}(\theta) = \frac{1}{\sqrt{N}}\left[1, e^{j\pi\theta}, \cdots, e^{j(N-1)\pi\theta}\right]^T, \tag{5}$$

where the spatial direction $\theta \in [-1, 1]$ is defined as $\theta = \sin\phi$, and $\phi \in [-\pi/2, \pi/2]$ is the physical direction.

However, in the XL-MIMO systems, the spherical wave propagation model should be used to characterize the near-field channel $\mathbf{h}_k^{\mathrm{near}}$ and it can be expressed as [8]

$$\mathbf{h}_k^{\mathrm{near}} = \sqrt{N}\alpha_0\mathbf{b}(\theta_0, r_0) + \sqrt{\frac{N}{L}}\sum_{l=1}^{L}\alpha_l\mathbf{b}(\theta_l, r_l), \tag{6}$$

where $\mathbf{b}(\theta, r)$ denotes the near-field beam steering vector, which is also called the near-field beam focusing vector [3]. For the ULA, the near-field beam focusing vector $\mathbf{b}(\theta, r)$ can be expressed as

$$\mathbf{b}(\theta, r) = \frac{1}{\sqrt{N}}\left[e^{-j\frac{2\pi}{\lambda}(r^{(0)}-r)}, \cdots, e^{-j\frac{2\pi}{\lambda}(r^{(N-1)}-r)}\right]^T, \tag{7}$$

where $r^{(n)}$ and r denotes the distance between the user and the nth BS antenna element and the center of the BS antenna, respectively. The distance $r^{(n)}$ can be written as

$$\begin{aligned} r^{(n)} &= \sqrt{r^2 - 2ndr\theta + n^2d^2} \\ &\overset{(a)}{\approx} r - nd\theta + \frac{n^2d^2}{2r}(1-\theta^2), \end{aligned} \tag{8}$$

where approximation (a) is the Fresnel approximation, which is derived by the second-order Taylor expansion $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \mathcal{O}(x^2)$. It can be obtained from (7) that the near-field channel is not only related to the angle of the user, but also to the distance between the user and the BS antenna.

Due to the near-field beamfocusing characteristic, the near-field NOMA systems are different from the far-field NOMA systems. In the near-field NOMA systems, the users located at the same direction but different distances can be divided into different groups rather than one group in the far-field NOMA systems, which is illustrated in Fig. 1. In the following section, the analysis and framework of near-field NOMA communications will be discussed.

III. Analysis of Two-User Near-Field NOMA Communications

In this section, we consider the simplified model of two-user near-field NOMA communications. Without loss of generality, we consider a downlink XL-MIMO communication system serving two users located at the same direction θ but different distances, where the user 1 is more closer to the BS than the user 2. For simplicity, we consider the single-path channel so the channel can be expressed as

$$\mathbf{h}_i = \sqrt{N}\alpha_i \mathbf{b}(\theta, r_i), \tag{9}$$

where $i \in \{1, 2\}$. For the near-field LDMA system, we suppose that the perfect CSI is acquired and the BS could get the optimal analog precoder $\mathbf{W}_{\mathrm{A}}$, i.e. $\mathbf{W}_{\mathrm{A}} = \mathbf{W} = [\mathbf{b}(\theta, r_1), \mathbf{b}(\theta, r_2)]$. For digital precoder, we adopt the ZF scheme. Then the achievable sum rate of the two users can be represented by the following lemma. It should be noted that when considering the normalized bandwidth, the spectrum efficiency is equivalent to the achievable sum rate.

Lemma 1. We assume that we adopt the single-path channel as (9) and the perfect CSI of the two users are acquired. By applying ZF digital precoding with equal power allocation, the achievable sum rate of the two users can be represented by

$$R_{\mathrm{LDMA}} = \sum_{i=1}^{2} R_k = \sum_{i=1}^{2} \log_2 \left(1 + \frac{P}{2\sigma^2} \frac{N|\alpha_i|^2}{[\mathbf{W}^H\mathbf{W}]^{-1}_{i,i}}\right), \tag{10}$$

where P denotes the transimit power at the BS and σ^2 is the noise power.

Proof. The proof is provided in [4] Appendix D. □

Based on (10), it can be observed that the correlation of channels has significant influence on the spectrum efficiency performance. The impact of channel correlation of two users on the spectrum efficiency can be characterized as follows.

Corollary 1 (The impact of channel correlation of two users on the spectrum efficiency of the near-field LDMA systems): With the channel correlation of two users μ as the variable, the spectrum efficiency of the two-user near-field LDMA systems can be represented as

$$R_{\mathrm{LDMA}} = \sum_{i=1}^{2} R_k = \sum_{i=1}^{2} \log_2 \left(1 + \frac{\beta N}{2} |\alpha_i|^2 (1 - \mu^2)\right), \tag{11}$$

where $\beta = \frac{P}{\sigma^2}$ denotes the transmit SNR.

Proof. The matrix $[\mathbf{W}^H\mathbf{W}]^{-1}$ can be expressed as

$$[\mathbf{W}^H\mathbf{W}]^{-1} = \frac{1}{\Delta} \begin{vmatrix} \mathbf{b}^H(\theta, r_1)\mathbf{b}(\theta, r_1) & -\mathbf{b}^H(\theta, r_1)\mathbf{b}(\theta, r_2) \\ -\mathbf{b}^H(\theta, r_2)\mathbf{b}(\theta, r_1) & \mathbf{b}^H(\theta, r_2)\mathbf{b}(\theta, r_2) \end{vmatrix} \tag{12}$$

where Δ denotes the determinant of the matrix $[\mathbf{W}^H\mathbf{W}]$ and it can be expressed as

$$\begin{aligned} \Delta &= 1 - \mathbf{b}^H(\theta, r_2)\mathbf{b}(\theta, r_1)\mathbf{b}^H(\theta, r_1)\mathbf{b}(\theta, r_2) \\ &= 1 - \left(\mathbf{b}^H(\theta, r_1)\mathbf{b}(\theta, r_2)\right)^H \mathbf{b}^H(\theta, r_1)\mathbf{b}(\theta, r_2) \\ &= 1 - \mu^2 \end{aligned} \tag{13}$$

By substituting (12) and (13) into (10), we can get (11) and the proof is completed.

□

For the near-field NOMA systems, the two users are simultaneously served by the same beam and let $\mathbf{w}$ denotes the analog precoding vector. Without loss of generality, we assume that $|\mathbf{h}_1^H\mathbf{w}|^2 \geq |\mathbf{h}_2^H\mathbf{w}|^2$. According to the principle of NOMA, we set α_1 and α_2 as the power allocation factor for user 1 and user 2, which satisfy $\alpha_1 = \alpha, \alpha_2 = 1 - \alpha$ and $\alpha_1 < \alpha_2$ [9].

If SIC can be successfully performed, then the user 1 can cancel the signal of user 2 and decodes its own signal. The achievable rate of the user 1 R_1 can be expressed as

$$R_1 = \log_2 \left(1 + |\mathbf{h}_1^H\mathbf{w}|^2 \alpha\beta\right), \tag{14}$$

and

$$R_2 = \log_2 \left(1 + \frac{|\mathbf{h}_2^H\mathbf{w}|^2 (1 - \alpha)\beta}{|\mathbf{h}_2^H\mathbf{w}|^2 \alpha\beta + 1}\right). \tag{15}$$

Then we can get the achievable sum rate of the two users in this near-field NOMA system $R_{\mathrm{NF-NOMA}}$, i.e., $R_{\mathrm{NF-NOMA}} = R_1 + R_2$.

To maximize the $R_{\mathrm{NF-NOMA}}$, we should adjust the power allocation factors and find the optimal solution. Additionally, R_1 and R_2 are usually set to be greater than the achievable rates of user 1 and user 2 in orthogonal multiple access (OMA) systems such as time division multiple access (TDMA), satisfying $R_i^{\mathrm{OMA}} = \frac{1}{2}\log_2\left(1 + |\mathbf{h}_i^H\mathbf{w}|^2\beta\right)$ where $i \in \{1, 2\}$. The optimal power allocation solution can be obtained by the following lemma.

Lemma 2. In a near-field NOMA system with two users, the optimal power allocation solution is

$$\alpha^* = \frac{1}{\sqrt{(1 + |\mathbf{h}_2^H\mathbf{w}|^2\beta)} + 1}. \tag{16}$$

Proof. The proof is provided in [10] Theorem 1. □

Therefore, by substituting (16) into $R_{\mathrm{NF-NOMA}}$, the optimal achievable sum rate of the two users in the near-field NOMA systems $R_{\mathrm{NF-NOMA}}^{\max}$ can be expressed as

$$R_{\mathrm{NF-NOMA}}^{\max} = \log_2 \left(\frac{{G_2}^2 + G_2{G_1}^2}{G_2 + 1}\right), \tag{17}$$

where $G_1 = \sqrt{1 + \beta|\mathbf{h}_1^H\mathbf{w}|^2}$, $G_2 = \sqrt{1 + \beta|\mathbf{h}_2^H\mathbf{w}|^2}$. Based on (17) , the impact of channel correlation of two users on the spectrum efficiency of the near-field NOMA systems can be analyzed and it can be derived as follows.

Corollary 2 (The impact of channel correlation of two users on the spectrum efficiency of the near-field NOMA systems):

We assume that the near-field beamfocusing vector focuses the energy on the location of user 1. The spectrum efficiency of the two-user near-field NOMA systems against μ can be represented as

$$R_{\mathrm{NF-NOMA}}^{\max} = \log_2\left(\frac{\left(G_2^{\mu}\right)^2 + G_2^{\mu}\left(G_1^{\mu}\right)^2}{G_2^{\mu}+1}\right), \tag{18}$$

where $G_1^{\mu} = \sqrt{1+\beta N|\alpha_1|^2}$, $G_2^{\mu} = \sqrt{1+\beta N|\alpha_2|^2\mu^2}$, and $\beta = \frac{P}{\sigma^2}$ denotes the transmit SNR.

Proof. According to the assumption, the analog precoding vector $\mathbf{w}$ can be expressed as $\mathbf{w} = \mathbf{b}(\theta, r_1)$. Then $|\mathbf{h}_1^H\mathbf{w}|^2$ and $|\mathbf{h}_2^H\mathbf{w}|^2$ can be expressed

$$|\mathbf{h}_1^H\mathbf{w}|^2 = |\sqrt{N}\alpha_1\mathbf{b}^H(\theta, r_1)\mathbf{b}(\theta, r_1)|^2 = N|\alpha_1|^2. \tag{19}$$

$$|\mathbf{h}_2^H\mathbf{w}|^2 = |\sqrt{N}\alpha_2\mathbf{b}^H(\theta, r_2)\mathbf{b}(\theta, r_1)|^2 = N|\alpha_2|^2\mu^2. \tag{20}$$

After substituting (19) and (20) into (17), we can get (18) and the proof is completed.

□

Therefore, we can compare the spectrum efficiency of the two-user near-field LDMA systems and near-field NOMA systems. Based on (11), we can easily derive that R_{LDMA} is an decreasing function of μ. For the near-field NOMA systems, we can also get that $R_{\mathrm{NF-NOMA}}$ is an increasing function of μ based on (18). There exists a boundary value μ^*, when the channel correlation of two users μ reaches μ^*, the spectrum efficiency of the two-user near-field LDMA and NOMA systems become equal, i.e., $R_{\mathrm{LDMA}} = R_{\mathrm{NF-NOMA}}$.

Thus, we can conclude that applying near-field NOMA can solve the problem of poor LDMA performance with medium channel correlation. The criteria for applying near-field NOMA based on channel correlation can be expressed as follows.

Remark 1 (Criteria for applying near-field NOMA based on channel correlation): For two-user near-field NOMA systems, if the channel correlation of two users μ is large than the boundary value μ^*, then we can apply near-field NOMA and it can achieve better spectrum efficiency performance than the near-field LDMA, i.e., $R_{\mathrm{NF-NOMA}} > R_{\mathrm{LDMA}}$. Besides, based on (17), no matter what value μ takes within the range $[0, 1]$, the spectrum efficiency of near-field NOMA systems $R_{\mathrm{NF-NOMA}}$ is always greater than 0, while the spectrum efficiency of near-field LDMA systems R_{LDMA} becomes 0 when μ is equal to 1.

So far, we have compared the spectrum efficiency of two-user near-field LDMA and NOMA systems and given the criteria for applying near-field NOMA. These analyses confirm the potential of applying near-field NOMA to mitigate multi-user interferences and enhance spectrum efficiency compared to near-field LDMA. Therefore, a framework of applying NOMA in near-field multi-user communication will be discussed in the following section.

IV. Framework of Multi-User Near-Field NOMA Communications

In this section, we extend the two-user near-field NOMA communications to the multi-user scenarios. As discussed in Section II, we consider a general multi-user near-field NOMA scheme where the number of beams and the number of users in each beams are arbitrary. Based on (3), our purpose is to maximize the overall spectrum efficiency of all K users. The optimization problem can be formulated as follows

$$\begin{aligned}
\max_{\{p_{m,n}, \mathbf{w}_{\mathrm{A},n}, \mathbf{w}_{\mathrm{D},n}\}} & \sum_{n=1}^{N_{\mathrm{RF}}}\sum_{m=1}^{|M_n|} R_{m,n} \\
\text{s.t.} \quad & C_1: \ p_{m,n} \geq 0, \quad \forall n, m, \\
& C_2: \ \sum_{n=1}^{N_{\mathrm{RF}}}\sum_{m=1}^{|M_n|} p_{m,n} \leq P, \\
& C_3: \ R_{m,n} \geq R_{\min}, \quad \forall n, m, \\
& C_4: \ |\mathbf{w}_{\mathrm{A},n}| = \frac{1}{\sqrt{N}} \\
& C_5: \ ||\mathbf{w}_{\mathrm{D},n}||_2 = 1
\end{aligned} \tag{21}$$

where $\mathbf{w}_{\mathrm{A},n}$ is the nth column of the analog precoder matrix $\mathbf{W}_{\mathrm{A}}$, P is the maximum transmit power at the BS, $R_{\min}$ is the minimum data rate for each user. The constraint C_1 and C_2 are the limitations of the transmitted power. The constraint C_3 represents that the date rate of every user must be larger than the minimum date rate $R_{\min}$. The constraint C_4 and C_5 are the analog precoder's constant-modulus constraint and the digital precoder's normalized constraint, respectively.

It can be seen that the optimization problem (21) is a non-convex problem. Therefore, a framework of multi-user near-field NOMA is proposed to solve the optimization problem (21), including the analog precoder and the digital precoder design and the dynamic power allocation algorithm.

For the analog precoder design, we can first design a polar-domain codebook according to **Algorithm 1** in [8]. Then we can perform the beam training procedure to select the optimal codeword for each user group, which can be expressed as $\mathbf{W}_{\mathrm{A}} = [\mathbf{w}_{\mathrm{A},1}, \mathbf{w}_{\mathrm{A},2}, \cdots, \mathbf{w}_{\mathrm{A},N_{RF}}]$. Then the equivalent channel vectors for all beams can be expressed as

$$\bar{\mathbf{h}}_{m,n}^H = \mathbf{h}_{m,n}^H\mathbf{W}_{\mathrm{A}}. \tag{22}$$

For the digital precoder design, as the channel correlation among users in different user groups is enough low, thus we adopt the zero-forcing (ZF) scheme. Specifically, the equivalent channel matrix for all beams can be expressed as

$$\bar{\mathbf{H}} = \left[\bar{\mathbf{h}}_{1,1}, \bar{\mathbf{h}}_{1,2}, \cdots, \bar{\mathbf{h}}_{1,N_{RF}}\right]^H, \tag{23}$$

where $\bar{\mathbf{h}}_{1,n}$ denotes the channel vector of user with strongest effective channel gain in *n*-th user group. Then the digital precoder matrix can be expressed as

$$\mathbf{W}_{\mathrm{D}} = \bar{\mathbf{H}}^H\left(\bar{\mathbf{H}}\,\bar{\mathbf{H}}^H\right)^{-1}\mathbf{\Lambda}, \tag{24}$$

where the diagonal matrix $\mathbf{\Lambda}$ is designed to normalize the precoder vectors and it satisfies $\|\mathbf{W}_{\mathrm{A}}\mathbf{w}_{\mathrm{D},n}\|_2^2=1$,

Based on the *Proposition 1* and the extension of the Sherman-Morrison-Woodbury formula in [11], the objective function in (21) can be further expressed as

$$R_{m,n}=\max_{c_{m,n}}\max_{a_{m,n}>0}\left(\log_2 a_{m,n}-\frac{a_{m,n}e_{m,n}}{\ln 2}+\frac{1}{\ln 2}\right), \tag{25}$$

where

$$\begin{aligned} e_{m,n} &= \mathrm{E}\left\{\left|s_{m,n}-c_{m,n}y_{m,n}\right|^2\right\} \\ &=\left|1-c_{m,n}\sqrt{p_{m,n}}\mathbf{h}_{m,n}^H\mathbf{w}_n\right|^2+|c_{m,n}|^2|\mathbf{h}_{m,n}^H\mathbf{w}_n|^2\sum_{i=1}^{m-1}p_{i,n} \\ &\quad+|c_{m,n}|^2\sum_{j\neq n}|\mathbf{h}_{m,n}^H\mathbf{w}_j|^2\sum_{i=1}^{|M_j|}p_{i,j}+|c_{m,n}|^2\sigma^2. \end{aligned} \tag{26}$$

Besides, we can substitute (2) and (3) into the constraint C_3 in (21) to transform it into linear constraint. Thus, the optimization problem (21) can be rewritten as

$$\begin{aligned} &\max_{\{p_{m,n}\}}\sum_{n=1}^{N_{\mathrm{RF}}}\sum_{m=1}^{|M_n|}\max_{c_{m,n}}\max_{a_{m,n}>0}\left(\log_2 a_{m,n}-\frac{a_{m,n}e_{m,n}}{\ln 2}+\frac{1}{\ln 2}\right) \\ &\text{s.t. } C_1\text{: } p_{m,n}\geq 0,\quad \forall n,m, \\ &\qquad C_2\text{: } \sum_{n=1}^{N_{\mathrm{RF}}}\sum_{m=1}^{|M_n|}p_{m,n}\leq P, \\ &\qquad C_3\text{: } \left|\mathbf{h}_{m,n}^H\mathbf{w}_n\right|^2p_{m,n}\geq(2^{R_{\min}}-1) \\ &\qquad\left(\left|\mathbf{h}_{m,n}^H\mathbf{w}_n\right|^2\sum_{i=1}^{m-1}p_{i,n}+\sum_{j\neq n}\left|\mathbf{h}_{m,n}^H\mathbf{w}_j\right|^2\sum_{i=1}^{|M_j|}p_{i,j}+\sigma^2\right) \end{aligned} \tag{27}$$

To solve the problem (27), we should iteratively optimize $c_{m,n}$, $a_{m,n}$, and $p_{m,n}$. Specifically, if the optimal power allocation $p_{m,n}^{t-1}$ in the $(t-1)$th iteration is obtained, then the $c_{m,n}^t$ and $a_{m,n}^t$ in the tth iteration can be expressed as

$$c_{m,n}^t=\frac{(\sqrt{p_{m,n}^{t-1}}\mathbf{h}_{m,n}^H\mathbf{w}_n)^*}{p_{m,n}^{t-1}|\mathbf{h}_{m,n}^H\mathbf{w}_n|^2+\chi_{m,n}^{t-1}}. \tag{28}$$

and

$$a_{m,n}^t=\frac{1}{1-\frac{|\mathbf{h}_{m,n}^H\mathbf{w}_n|^2p_{m,n}^{t-1}}{|\mathbf{h}_{m,n}^H\mathbf{w}_n|^2p_{m,n}^{t-1}+\chi_{m,n}^{t-1}}}, \tag{29}$$

where

$$\chi_{m,n}^{t-1}=|\mathbf{h}_{m,n}^H\mathbf{w}_n|^2\sum_{i=1}^{m-1}p_{i,n}^{t-1}+\sum_{j\neq n}|\mathbf{h}_{m,n}^H\mathbf{w}_j|^2\sum_{i=1}^{|M_j|}p_{i,j}^{t-1}+\sigma^2 \tag{30}$$

After obtaining the optimal $c_{m,n}^t$ and $a_{m,n}^t$ in the t-th iteration, the problem of obtaining the optimal $p_{m,n}^t$ becomes a convex optimization problem and we can solve it through numerical convex software. Therefore, by iteratively updating $c_{m,n}^t$, $a_{m,n}^t$, and $p_{m,n}^t$, we can obtain the final stationary solution to the optimization probelm (27).

The overall framework of multi-user near-field NOMA communication systems is summarized in **Algorithm 1**.

Algorithm 1 Overall Framework of Multi-Users NF-NOMA

Inputs: the multi-users channel matrix $\mathbf{H}$.
Output: the analog precoder $\mathbf{W}_{\mathrm{A}}$, the digital precoder $\mathbf{W}_{\mathrm{D}}$, the power allocation $p_{m,n}$.
Analog Precoder Design:
Design the near-field polar-domain codebook $\mathcal{W}$;
BS perform the beamtraining procedure and get the analog precoder as $\mathbf{W}_{\mathrm{A}}=[\mathbf{w}_{\mathrm{A},1},\mathbf{w}_{\mathrm{A},2},\cdots,\mathbf{w}_{\mathrm{A},N_{RF}}]$;
Digital Precoder Design:
Get the digital precoder $\mathbf{W}_{\mathrm{D}}$ according to (24).
Dynamic Power Allocation Algorithm:
Solve the problem (27) by iteratively optimizing $c_{m,n}$, $a_{m,n}$, and $p_{m,n}$ and get the power allocation $p_{m,n}$.
return $\mathbf{W}_{\mathrm{A}}$, $\mathbf{W}_{\mathrm{D}}$ and $p_{m,n}$.

V. Simulation Results

In this section, simulation results are provided to verify the performance of near-field NOMA system. Specifically, the BS equipped with 512-element ULA serves K users with single-antenna. We consider the linear distributed scenario, where the users are randomly located at the same angle but different distances within $[4\text{ m}, 100\text{ m}]$. Besides, the frequency is 30 GHz and the spacing of the arrays is $d=\frac{\lambda}{2}=0.5$ cm.

In addition, some benchmark comparison schemes are considered as follows: (1) Fully-digital, where each antenna is equipped with one RF chain, i.e., $N_{\mathrm{RF}}=N$; (2) Near-field LDMA [4], where the near-field polar-domain codebook is applied and each beam only serves one user, i.e., $N_{\mathrm{RF}}=K$; (3) Far-field SDMA, where the far-field DFT codebook is applied and each beam only serves one user, i.e., $N_{\mathrm{RF}}=K$.

In Fig. 2, the spectrum efficiency of multi-user near-field NOMA systems against SNR is shown, where $K=4$, $N_{RF}=\frac{K}{2}$ and $R_{min}=1$ bps/s/Hz. It can be shown that the proposed near-field NOMA systems can achieve the best spectrum efficiency compared to the near-field LDMA and the far-field SDMA systems. This is because that the near-field NOMA systems can utilize the near-field beamfocusing characteristic and apply dynamic power allocation, which mitigates inter-group and intra-group interferences.

Next, the spectrum efficiency of multi-user near-field NOMA systems against the number of users K is shown in Fig. 3, where SNR = 20 dB. It can be shown that the spectrum of near-field NOMA systems gradually increases with the increasing of K. The reason is that as K increases, the probability of different users being assigned to the same user group increases and more users can benefit from the spectrum efficiency improvement brought by near-field NOMA.

Furthermore, the spectrum efficiency of multi-user near-field NOMA systems against the minimum date rate R_{min} is shown in Fig. 4, where SNR = 20 dB. It is shown that the spectrum efficiency of near-field NOMA systems decreases with the increasing of R_{min}. This is because a higher R_{min} means that the system needs to sacrifice some transmission power

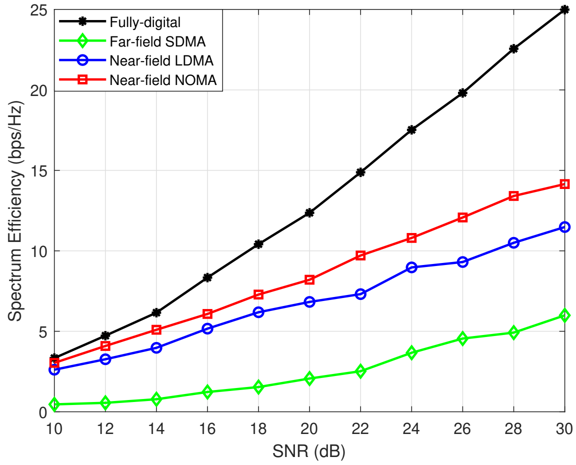

Fig. 2. Spectrum efficiency of multi-user near-field NOMA systems against SNR.

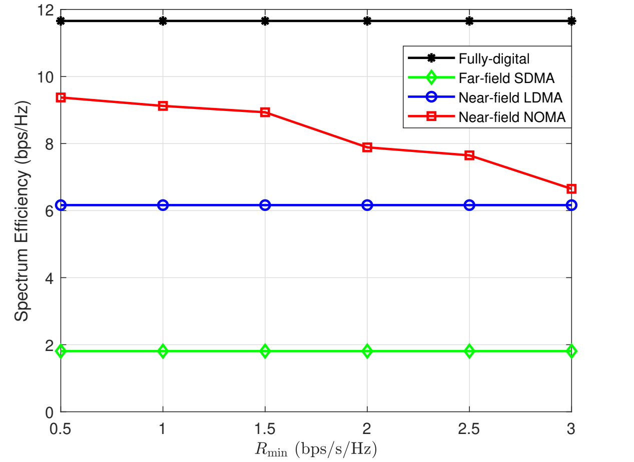

Fig. 4. Spectrum efficiency of multi-user near-field NOMA systems against the minimum date rate R_{min}.

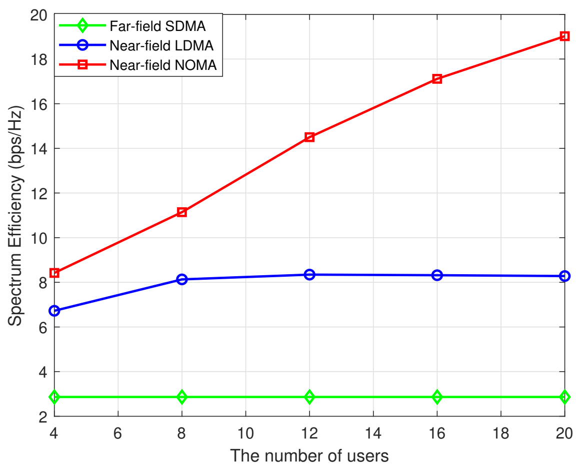

Fig. 3. Spectrum efficiency of multi-user near-field NOMA systems against the number of users K.

to ensure that low QoS users reach R_{min}, which leads to a decrease in the overall spectrum efficiency performance of the system.

VI. Conclusions

In this paper, near-field NOMA is applied in near-field communications to further enhance spectrum efficiency. The closed-form solutions of the spectrum efficiency of two-user near-field NOMA systems are analysed, formulating the criteria for applying near-field NOMA. Then, the near-field NOMA communications is extended to multi-user scenarios and the overall framework of applying NOMA in near-field multi-user communications is provided. Simulation results verify the superiority of near-field NOMA on spectrum efficiency over near-field LDMA. The proposed near-field NOMA communications can be considered as a effective way to further enhance spectrum efficiency, enabling more users to benefit from near-field communications.

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